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Provincial Department of Education - NWP

තෙවන වාර පරීක්ෂණය - 13 ශ්‍රේණිය - 2023

Third Term Test - Grade 13 - 2023

විභාග අංකය: .....

Combined Mathematics - I

Time: 03 Hours

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மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer decide which of them you will prioritise.

Index number							

Instructions:

* *This question paper consists of two parts,* **Part A** (Questions 1 - 10) and **Part B** (Questions 11 - 17)
* **Part A:** *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:** *Answer **five** questions only. Write your answers on the sheets provided*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on the top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall*

For Examiners' Use only

(10) Combined Mathematics I		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	

Total

In numbers	
In words	

Part A

01. Using the principle of mathematical induction, prove that $n^3 - n$ is divisible by 6 for all $n \in Z^+$ and $n \geq 2$.

02. Sketch the graph of $y = |x + 2|$ and $y = 7 - |x - 3|$ in the same diagram. Hence or otherwise, find all real values of x satisfying the inequality $|x + 1| + |x - 4| < 7$.

03. On an same Argand diagram, sketch the locus representing complex numbers z satisfying $|z+i|=1$ and the locus representing complex number w satisfying $\arg(w-2)=\frac{3\pi}{4}$. Hence, find the least value of $|z-w|$ for the points on these loci.

04 . Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(2+kx)^7$ where k is a constant. Give each term in its simplest form.

Given that the coefficient of x^3 in this expansion is 1890, find the value of k.

05. Evaluate $\lim_{x \longrightarrow 0} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$.

06. Show that the area of the region enclosed by the curves $y = e^x$, $y = x^2 - 1$, $x = -1$ and $x = 1$ is $\frac{3e^2 + 4e - 3}{3e}$.

07. A curve is given parametrically by $x = 4t - 1,\ y = \frac{5}{2t} + 10,\ \ t \in R,\ \in t \neq 0$. The curve crosses the x - axis at the point A. Find the coordinates of A. Show that an equation of the tangent to the curve at A is $y + 10x + 20 = 0$.

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08. A line intersects the x - axis at $A(7,0)$ and the y - axis at $B(0,-5)$. A variable line PQ is drawn perpendicular to AB intersecting the x - axis in P and the y - axis in Q. If AQ intersects BP at R, then find the locus of R.

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09. A variable tangent line drawn to a circle whose radius is c and the centre is the origin, meets the x - axis and y - axis at A and B respectively. Find the locus of the centre of the circle passing through O, A and B.

10. Solve the equation $\sin^{-1} x - \cos^{-1} x = \sin^{-1}(3x-2)$.

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තෙවන වාර පරීක්ෂාය - 13 ශ්‍රේණිය - 2023

Third Tem Test - Grade 13 - 2023

Combined Mathematics - I

Part B

❖ *Answer only five questions.*

11. *a.* The roots of the quadratic equation $x^2 - x + p = 0$ are α and β. Also the roots of $x^2 - 9x + q = 0$ are γ and δ.

If α, β, γ and δ are in a geometric progression, then find the possible values of common ratio of that progression.

Then find the possible values of p and q.

Obtain the quadratic equations whose roots are $\alpha\gamma$ and $\beta\delta$.

b. Remainder when the polynomial $g(x)$ of degree 3 is divided by $x, (x-1), (x+1)$ and $(x-2)$ are $-12, -8, -24$ and -6 respectively. Given that $Q(x) = (x+2)g(x) + 24$ show that $x, (x-1), (x+1)$ and $(x-2)$ are the factors of $Q(x)$. Hence find $Q(x)$ without finding $g(x)$.

12. *a.* A cricket team consisting of 11 players is to be formed from 16 players of whom 4 can be bowlers and 2 can keep wicket and the rest can neither be bowler nor keep wicket. In how many different ways can a team be formed so that the teams contain

i. exactly 3 bowlers and 1 wicket keeper,

ii. at least 3 bowlers and at least 1 wicket keeper?

b. U_r is the r^{th} term of the sequence $\frac{1}{2}, \frac{1\cdot 3}{2\cdot 4}, \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}, \frac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6\cdot 8}, \ldots$

Express $U_{(r+1)}$ in terms of U_r.

$f(r)$ is a function of r, where $f(r)=(Ar+B)U_r$; A and B are constants and $f(r+1)-f(r)=Ur$. Find the values of A and B

and **hence**, prove that $\sum_{r=1}^{n} U_r = \left[\frac{1\cdot 3\cdot 5\cdot 7.....(2n+1)}{2\cdot 4\cdot 6.....2n}-1\right]$.

13. *a.* Let $A\equiv\begin{pmatrix}2 & a\\ 3 & b\end{pmatrix}$, $B\equiv\begin{pmatrix}1\\ 1\end{pmatrix}$ and $C=\begin{pmatrix}1\\ 3\end{pmatrix}$, where $a,b\in\Re$.

It is given that $AB=C$. Show that $a=-1$ and $b=0$. Then show that $A^2=2A-3I$.

Deduce that $A^3=A-6I$

The inverse of A is denoted by A^{-1} show that $A^{-1}=\frac{1}{3}(2I-A)$. Find A^{-1}

b. i. The complex number u is defined by $u=\frac{(1+2i)^2}{2+i}$.

Express u in the form $x+iy$, where x and y are real.

Sketch an Argand diagram showing the locus of the complex number z such that $|z-u|=|u|$

ii. Find the square roots of the complex number $7-6\sqrt{2}i$. Give your answers in the form $x+yi$. Where x and y are real numbers.

iii. Find the argument and the modulus of $2\sqrt{3}-2i$.

Find all the solutions z to the equation $z^3=2\sqrt{3}-2i$.

14. *a.* Let $f(x)=\frac{(x+1)(x-2)}{(x-1)^2}$ for $x\neq 1$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x)=\frac{(-x+5)}{(x-1)^3}$ for $x\neq 1$.

Hence, find the interval on which $f(x)$ is increasing and the interval on which $f(x)$ is decreasing. Also find the coordinates of the turning point of $f(x)$.

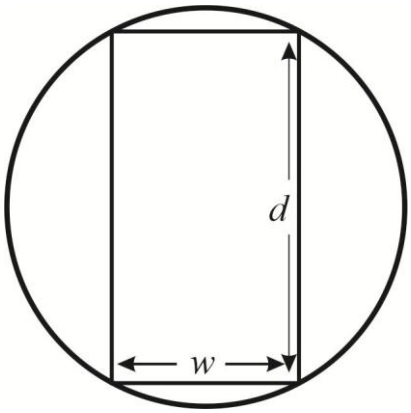
It is given that $f''(x)=\dfrac{2(x-7)}{(x-1)^4}$ for $x\neq 1$.

Find the coordinates of the points of inflection of the graph of $y=f(x)$.

Sketch the graph of for $y=f(x)$ indicating the asymptotes, the turning points and the points of inflection.

b. The strength of a beam with rectangular cross section is proportional to the product of its width w and the square of its depth d.

Find the dimensions of the strongest beam that can be cut from a cylindrical log of radius r in terms of r, considering the given figure which depicts the cross section of the log.



15. *a*. Find the value of the constants A, B and C such that

$\dfrac{1}{x(x^2+1)}=\dfrac{A}{x}+\dfrac{Bx+C}{x^2+1}$. Hence find the integrate $\int\dfrac{1}{x(x^2+1)}dx$

Use the substitution $x=\cos\theta$ show that $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{2\sin\theta}{\cos\theta+\cos^3\theta}d\theta=\ln\left(\frac{5}{3}\right)$

b. Find the value of $\int e^{-2x}\sin\pi x dx$ using the integration by parts.

c. Show that $\displaystyle\int_0^{\pi} f(x)\,dx=\int_0^{\pi} f(\pi-x)\,dx$.

If $I=\displaystyle\int_0^{\pi} x\sin^3 x dx$, using the above result, show that $I=\dfrac{1}{2}\displaystyle\int_0^{\pi}\sin^3 x dx$

Hence, find the exact value of the integral $\displaystyle\int_0^{\pi} x\sin^3 x dx$.

16. Let the equations of two circles be $x^2+y^2+2gx+2fy+c=0$ and $x^2+y^2+2g'x+2f'y+c'=0$.

If these circles intersect orthogonally, then show that $2gg'+2ff'=c+c'$.

Let u_1 and u_2 be two parallel lines passing through the points $P\equiv(1,0)$ and $Q\equiv(2,0)$ respectively. Let the line $2y-3x+7=0$ meet u_1 at A and u_2 at B.

If the length of AB is $\sqrt{13}$ units and gradient is positive, find the points $A(a,b)$ and $B(c,d)$. Where $a,b,c,d\in Z$

The vertices of a triangle are A, B and $C(6,1)$. Find the coordinates of the orthocenter. Find the equations of the circles whose diameters are AH and BC and show that those circles intersect each other orthogonally.

17. *a.* Show that $\sin^2 5^\circ+\sin^2 10^\circ+\sin^2 15^\circ+...\ +\sin^2 90^\circ=9\frac{1}{2}$.

b. State and prove the **cosine rule** for any triangle ABC in the usual notation.

For any triangle ABC in the usual notation, if $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$, then find the ratio $\cos A:\cos B:\cos C$.

c. In a right angled triangle, the hypotenuse is $2\sqrt{2}$ times the length of perpendicular drawn from the opposite vertex of the hypotenuse. Then find the other two angles.

d. Solve the equation $81^{\sin^2 x}+81^{\cos^2 x}=30$ in the interval $[0,\pi]$.