Provincial Department of Education - NWP

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Third Term Test - Grade 12 - 2023

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Index No. : ....

Combined Mathematics - I

Time : 03 hours

Additional time for reading 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer decide which of them you will prioritise.

Index number							

Instructions:

- *This question paper consists of two parts*
  Part A (Questions 1 - 10) and **Part B** (Questions 11 - 17)
- **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
- **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided*
- *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on the top of* **Part B** *and hand them over to the supervisor.*
- *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall*

For Examiners' Use only

(10) Combined Mathematics I		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	

Total

In numbers	
In words	

Part A

(01) Sketch the rough graphs of $y = |x - 1|$ and $y = 1 - x^2$ in the same diagram. Hence or otherwise, find all real values of x satisfying the inequality $2|x - 2| < 4 - x^2$.

(02) Solve the equation $\text{Log}_{1/2}(2x - 1) + \log_{2x-1} 4 = 1 \ (x > ½)$.

(03) $$\lim_{x \to 2} \frac{\sqrt{2x-1} - \sqrt{x+1}}{\sin \frac{\pi x}{2}} = -\frac{1}{\sqrt{3}\,\pi}$$

(04) The remainders, when the polynomial $f(x) = x^3 + px^2 + qx - 6$ is divided by $(x-1)$ and $(x+2)$ are equal. Show that $p - q = 3$. Also if it is given that $(x-2)$ is a factor of $f(x)$, obtain the value of P.

(05) Determine the values of A, B and C such that $4x^2 - x - 1 = A(x^2 - 3x + 2) + B(x - 2) + C(x - 1)$

Hence or otherwise, find the partial fractions of $\dfrac{4x^2 - x - 1}{(x - 2)(x - 1)}$.

(06) The point P divides the line BC in the ratio 1:2 internally. If $B \equiv (-1, 1)$ and $C \equiv (1, 4)$, write the equation of the straight line passing through P and the point $(2, 1)$.

(07) A curve C is parametrically given by $x = (t + 1)^3$ and $y = t^3 + 1$. Write the equation of the tangent and the normal line drawn to the curve C at the point relevant to $t = 2$.

(let the gradient of the tangent x gradient of the normal = -1)

(08) Write the set of values of λ such that the roots of the quadratic equation

$(1 + 2\lambda)x^2 - 10x + \lambda - 2 = 0, \quad \lambda \in \mathbb{R} - \{-\frac{1}{2}\}$ are real.

(09) If $\frac{b+c}{5} = \frac{c+a}{6} = \frac{a+b}{7}$ according to the standard notation, using the sine rule for a triangle, show that $\frac{\text{Sin A}}{4} = \frac{\text{Sin B}}{3} = \frac{\text{Sin C}}{2}$

(10) Find the general solutions of the equation

$(\sqrt{3} - 1)\ \text{Sin}^2 x - (\sqrt{3} + 1)\ \text{Sin}\ x\ \text{Cos}\ x + 1 = 0$

(11) (a) The roots of the quadratic equation $px^2 + qx + r = 0$ $(p, q, r, \in \mathbb{R})$ are α and β. Write the values of $\alpha + \beta$ and $\alpha\beta$ in terms of p, q and r.

If $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$, show that $2p^2r = r^2q + q^2p$

Also show that the quadratic equation with roots

$\sqrt{\frac{\alpha}{\beta}}$ and $\sqrt{\frac{\beta}{\alpha}}$ is $\sqrt{pr}\,x^2 + qx + p = 0$

(b) Let $\frac{x^2 - bx}{ax + c} = \frac{k - 1}{k + 1}$ where $a > b$ and $a, b, c \in \mathbb{R}$.

Express the above quadratic equation in the from $AX^2 + BX + C = 0$. If the roots of this equation are equal in magnitude and opposite in direction show that $k = \frac{a - b}{a + b}$.

Also if the roots of that quadratic equation are real, find the range of values of K.

(c) The remainder, when the polynomial function $f(x) = x^3 - 3x^2 + kx + 8$ where $k \in \mathbb{R}$ is divided by $x - 2$, is 2. Find the value of k. For that value of k, if $g(x) = x^2 - x - 4$, find a function $p(x)$, such that $p(x) = f(x) + g(x)$. Show that $(x - 2)$ is a factor of $p(x)$.

Find λ, μ such that $p(x) = (x - 2)[(x - 2)^2 + \lambda(x-2) + \mu]$. Hence **deduce that** the remainder, when $p(x)$ is divided by $(x - 2)^2$

(12) (a) Sketch the rough graph of the function $f(x) = \frac{x^2 - 4}{|x - 2|}$

Hence show that $\lim\limits_{x \to 2} \frac{x^2 - 4}{|x - 2|}$ does not exist.

(b) Show that $\lim\limits_{x \to \frac{\pi}{6}} \frac{(8\,\text{Sin}^3 x - 1)(2\,\text{Cos}\, x - \sqrt{3})}{(x - \frac{\pi}{6})^2} = -3\sqrt{3}$

(c) Sketch the rough graph of $y = x(x - 4)$. Hence deduce the graph of $y = |x(x - 4)|$ in a seperate coordinate plane. Also sketch the rough graph of $y = 4 - 2|x - 2|$ in the same coordinate plane and **deduce that** $|x(x - 4)| > 4 - 2|x - 2|$ for all $x \in \mathbb{R}$ except for three values of x. Write these three special values also.

(13) (a) Prove that $\text{Log}_a b = \dfrac{\text{Log}_c b}{\text{Log}_c a}$ where $a, b, c \in \mathbb{R}^+$

Hence **deduce that** $\text{Log}_{a^k} b = \dfrac{1}{k} \text{Log}_a b$

Hence or otherwise, Show that

$$\text{Log}_2 x + \text{Log}_{2^2} x^2 + \text{Log}_{2^3} x^3 + \ldots\ldots\ldots\ldots + \text{Log}_{2^n} x^n = \text{Log}_2 x^n$$

(b) Find the domain and the range of the function $f(x) = \sqrt{x^2 - 4x - 5}$.

If there exist a function $g(x) = x - 3$, $g : \mathbb{R} - (2, 8) \longrightarrow \mathbb{R} - (-1, 5)$, write $f \circ g$. Hence **deduce that** domain of the function $f \circ g$.

(c) Find the partial fractions of the function $\dfrac{x^2}{(x+1)(x+2)}$

Find λ, μ and γ such that $x^2 = \lambda(x+1)(x+2) + \mu(x+1) + \gamma$.

Hence deduce the partial fractions of $\dfrac{1}{(x+1)(x+2)}$.

(14) (a) Using the first principles, show that the first derivative of $\sin x$ is $\text{Cos}\, x$.

If $y = \dfrac{\text{Sin}\, x - \text{Cos}\, x}{\text{Sin}\, x + \text{Cos}\, x}$, Show that $\dfrac{dy}{dx} = 1 + y^2$

find $\dfrac{d^2y}{dx^2}$ and show that $\dfrac{d^2y}{dx^2} = 0$ if and only of $y = 0$.

show that $\dfrac{d^2y}{dx^2} = 8(2 - \sqrt{3})^2$, when $x = \dfrac{\pi}{3}$

(b) A curve is parametrically given by $x = a\cos^2\theta$ and $y = b\sin\theta$ where $(\theta \neq n\pi,\ n \in \mathbb{Z})$.

Differentiating the parametric functions show that $\dfrac{dx}{d\theta} = \dfrac{-2ay}{b^2} \dfrac{dy}{d\theta}$

Hence show that $\dfrac{dy}{dx} = \dfrac{-b^2}{2ay}$.

Also obtain that $\dfrac{d^2y}{dx^2} = \dfrac{b^2}{2ay^2} \dfrac{dy}{dx} = \dfrac{-b^4}{4a^2y^3}$

Show that $\dfrac{d^2y}{dx^2} = \dfrac{-b}{4a^2}$ when $\theta = \dfrac{\pi}{2}$

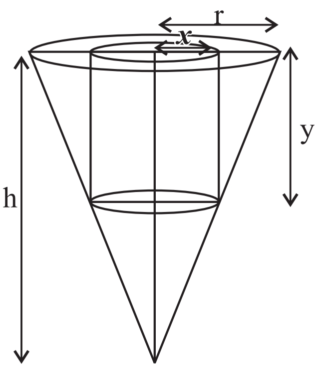
(15) (a) Let $f(x) = \dfrac{x^2}{(x-1)^2}$ for $x \neq 1$. Using the standard notation show that

$f'(x) = \dfrac{-2x}{(x-1)^3}$ for $x \neq 1$

Also it is given that $f''(x) = \dfrac{2(2x+1)}{(x-1)^4}$, $x \neq 1$

find the turning points of the graph of $y = f(x)$ by finding the sign of $f'(x)$. Find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing. Finding the sign of $f''(x)$, find the coordinates of the inflection points of the graph of $y = f(x)$. Sketch the graph of $y = f(x)$ indicating the asymptotes, turning points and the point of inflection. Using the graph or otherwise find the intervals of values of k for the quadratic equation $(1-k)x^2 + 2kx - k = 0$ to have two distinct real roots. Here $k \in \mathbb{R}$.

(b) A cylinder with radius x and height y is kept inside a hollow cone of radius r and height h. so that the two axes coincide each other as shown in the figure.



Show that $y = \dfrac{h(r-x)}{r}$ and show that the volume of the cylinder $v = \dfrac{\pi h}{r} x^2(r-x)$. By examining the sign of $\dfrac{dv}{dx}$, show that the value of x such that the volume (v) of the cylinder is maximum is $\dfrac{2r}{3}$. Also show that the ratio between the volume of the cone and the volume of the cylinder is 9:4.

(16) (a) For a triangle ABC, using the standard notation, state the cosine rule.

In a triangle ABC, the lengths of the sides BC, CA and AB are $x + y$, x and $x - y$ respectively. Show that $\text{Cos } A = \dfrac{x - 4y}{2(x - y)}$. If the magnitude of the angle A is 120°, show that $5y = 2x$. Using the sine rule **deduce** that $\dfrac{5 \text{ Sin } A}{7} = \dfrac{\text{Sin } B}{1} = \dfrac{5 \text{ Sin } C}{3}$

(b) Prove that $\sin(A+B) - \text{Sin}(A-B) = 2 \text{ Cos} A \text{ Sin} B$

Hence deduce that $\text{Cos}(A+B) - \text{Cos}(A-B) = -2 \text{ Sin} A \text{ Sin} B$

Show that $\text{Sin}(60-x)\text{Sin}(60+x) = \dfrac{1}{2}\text{Cos } 2x + \dfrac{1}{4}$ and hence find the value of λ such that $\text{Sin } x \text{ Sin}(60-x)\text{ Sin}(60+x) = \lambda \text{ Sin } 3x$. **Hence** find the general solutions of the equation $\text{Sin } x \text{ Sin}(60-x)\text{ Sin}(60+x) = \dfrac{1}{8}$

(c) Solve the equation $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\dfrac{8}{31}\right)$

(17) (a) Let $f(x) = 4(\sin^4 x + \cos^4 x)$.

Find the values of A and B such that $f(x) = A + B\cos 4x$, $A, B \in \mathbb{R}$

Hence **deduce that** $2 \leqslant f(x) \leqslant 4$. Also sketch a rough graph of $y = f(x)$ in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. **Hence or otherwise** find the values of k or interval of values of k for the equation Cos $4x = k$ to have

i) two real roots

ii) three real roots

iii) four real roots within the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(b) Prove that the identity $2\cos^2 x - 2\cos^2 2x = \cos 2x - \cos 4x$.

Hence show that $\cos\frac{\pi}{5} - \cos\frac{2\pi}{5} = \frac{1}{2}$

Hence obtain that $\cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$.