දකුණු පළාත් අධ්‍යාපන දෙපාර්තමේන්තුව

සංයුක්ත ගණිතය - 12 ශ්‍රේණිය

දෙවන වාර පරීක්ෂණය - 2019

ලකුණු දීමේ පටිපාටිය

01) $f: A \longrightarrow B$ $\quad \therefore y = f(x) = x^2 + x \;;\; x = \{x \in \mathbb{R},\ x \geq -\frac{1}{2}\}$ හා

$y = \{y \in \mathbb{R},\ y \geq -\frac{1}{4}\}$ වේ.

$f(x_2) = x_2^2 + x_2 \quad , \quad f(x_1) = x_1^2 + x_1$

$x_2^2 + x_2 = x_1^2 + x_1$

$(x_2^2 - x_1^2) + (x_2 - x_1) = 0$

$(x_2 - x_1)(x_2 + x_1 + 1) = 0$

$\therefore (x_2 - x_1) = 0$ හෝ $(x_2 + x_1 + 1) = 0$

$x_2 = x_1$ හෝ $x_2 = -(x_1 + 1)$ (05)

$x_2 = -(x_1 + 1)$ සලකමු.

$x_1 \geq -\frac{1}{2}$ විට $x_2 \leq -\frac{1}{2}$ වේ. නමුත් $x \in \mathbb{R},\ x \geq -\frac{1}{2}$ බැවින්

$x_2 \neq -(x_1 + 1)$ වේ. (05)

$\therefore f(x_2) = f(x_1)$ වන්නේ $x_2 = x_1$ විට පමණි. (05)

$\therefore$ <u>f එකට - එක ශ්‍රිතයකි.</u>

$y = x^2 + x$ ලෙස ගනිමු.

එවිට, $x^2 + x - y = 0$ වේ.

$\therefore x = \frac{-1 \pm \sqrt{1 + 4y}}{2}$ (05) ; $y \geq -1/4$ වන සියලු y අගයන් සඳහා

x පවතී. $\therefore$ <u>f මතට ශ්‍රිතයකි.</u>

$\boxed{25}$

$A \equiv (3, 4) \quad , \quad B \equiv (-2, -4) \quad , \quad P \equiv (x, y)$

$AP^2 = [(x-3)^2 + (y-4)^2]$ (05) ; $BP^2 = [(x+2)^2 + (y+4)]^2$ (05)

$\frac{AP}{BP} = 2 \quad \therefore AP^2 = 4BP^2$ (05)

$(x-3)^2 + (y-4)^2 = 4[(x+2)^2 + (y+4)^2]$ (05)

$x^2 - 6x + 9 + y^2 - 8y + 16 = 4x^2 + 16x + 16 + 4y^2 + 32y + 64$

<u>$3x^2 + 3y^2 + 22x + 40y + 55 = 0$</u> (05)

$\left(x^2 + y^2 + \frac{22}{3}x + \frac{40}{3}y + \frac{55}{3} = 0\right)$ යන්න P ලක්ෂ්‍යයේ

පථය වේ. $\boxed{25}$

(03) $\dfrac{x-1}{(x^2+1)(x-3)} \equiv \dfrac{A}{x-3} + \dfrac{Bx+C}{x^2+1}$ 05

$x-1 \equiv A(x^2+1) + (Bx+C)(x-3)$

සංගුණක සැසඳීමෙන්,

$\boxed{x^2} \Rightarrow A+B = 0$ —— ①

$\boxed{x^1} \Rightarrow C-3B = 1$ —— ②

$\boxed{x^0} \Rightarrow A-3C = -1$ —— ③

$A = \dfrac{1}{5}, \ B = \dfrac{-1}{5}, \ C = \dfrac{2}{5}$ 05

$\therefore \dfrac{x-1}{(x^2+1)(x-3)} = \dfrac{1}{5(x-3)} - \dfrac{(x-2)}{5(x^2+1)}$ 05

$x = y+1$ විට ඉහත ප්‍රකාශනය $\dfrac{y}{(y^2+2y+2)(y-2)}$ ලැබේ.

$\therefore \dfrac{y}{(y^2+2y+2)(y-2)} = \dfrac{1}{5(y+1-3)} - \dfrac{(y+1-2)}{5[(y+1)^2+1]}$ 05

$= \dfrac{1}{5(y-2)} - \dfrac{(y-1)}{5(y^2+2y+2)}$ 05

25

(04)

(a) $\lim_{x \to 2} \dfrac{\sqrt{4-x^2}}{\sqrt{6-5x+x^2}}$

$= \lim_{x \to 2} \dfrac{\sqrt{(2-x)(2+x)}}{\sqrt{(2-x)(3-x)}}$ 05

$= \lim_{x \to 2} \sqrt{\dfrac{2+x}{3-x}}$

$= \sqrt{4}$

$= 2$ 05

(04) (b) $\lim_{x \to 0} \dfrac{a - \sqrt{a^2 - x^2}}{x^2}$

$= \lim_{x \to 0} \dfrac{(a - \sqrt{a^2 - x^2})(a + \sqrt{a^2 - x^2})}{x^2 (a + \sqrt{a^2 - x^2})}$ 05

$= \lim_{x \to 0} \dfrac{a^2 - (a^2 - x^2)}{x^2 (a + \sqrt{a^2 - x^2})}$

$= \lim_{x \to 0} \dfrac{1}{a + \sqrt{a^2 - x^2}}$ 05

$= \underline{\dfrac{1}{2a}}$ 05

$\boxed{25}$

(05) $\log_{10}(x-14) + \log_{10}(x-5) = 1$

$\log_{10}(x-14)(x-5) = 1$ 05

$(x-14)(x-5) = 10^1$ 05

$x^2 - 19x + 60 = 0$

$(x-4)(x-15) = 0$

$x = 4$ හෝ $x = 15$ 05

$x = 4$ විට, $\log_{10}(x-14) + \log_{10}(x-5)$ අර්ථ දක්වා නොමැත.

$\therefore \underline{x = 15}$ වේ. 05

$\boxed{25}$

(06)

(a) $\sec^2\theta (1 + \sec 2\theta) = 2\sec 2\theta$

$L.H.S = \sec^2\theta (1 + \sec 2\theta)$

$= \sec^2\theta \left(1 + \dfrac{1}{\cos 2\theta}\right)$

$= \dfrac{1}{\cos^2\theta} \cdot \dfrac{(\cos 2\theta + 1)}{\cos 2\theta}$

$= \dfrac{2\cos^2\theta}{\cos^2\theta \cdot \cos 2\theta}$ 05 $= 2 \cdot \dfrac{1}{\cos 2\theta}$

$= \underline{2\sec 2\theta = R.H.S}$ 05

(06)

(b) $\operatorname{cosec}\theta + 2\operatorname{cosec}2\theta \equiv \sec\theta \cdot \cot\frac{\theta}{2}$

$L.H.S = \operatorname{cosec}\theta + 2\operatorname{cosec}2\theta$

$= \frac{1}{\sin\theta} + \frac{2}{\sin 2\theta}$ (05)

$= \frac{\cos\theta + 1}{\sin\theta\cos\theta}$ (05)

$= \frac{2\cos^2\theta/2}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} \cdot \frac{1}{\cos\theta}$ (05)

$= \cot\theta/2 \cdot \sec\theta = R.H.S$

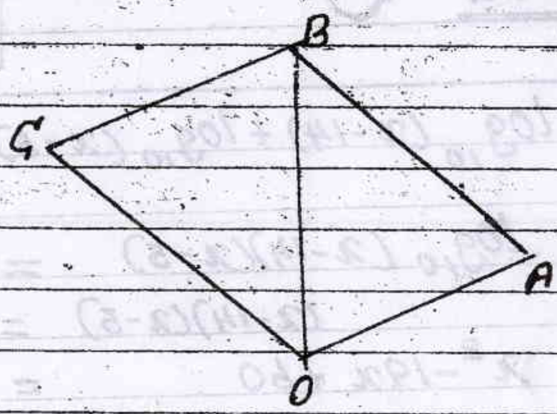
$\boxed{25}$

(07)

(a) $\overrightarrow{OA} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$

$\overrightarrow{OB} = 8\underset{\sim}{j}$

$\overrightarrow{OC} = -3\underset{\sim}{i} + 4\underset{\sim}{j}$



$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$\overrightarrow{AB} = -3\underset{\sim}{i} + 4\underset{\sim}{j}$ (05)

$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = -3\underset{\sim}{i} - 4\underset{\sim}{j}$ (05)

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = -6\underset{\sim}{i}$ (05)

(b) $\overrightarrow{OB} \cdot \overrightarrow{AC} = (8\underset{\sim}{j}) \cdot (-6\underset{\sim}{i}) = 0$ (05)

$\therefore$ $OB \perp AC$ වේ.

තවද $\overrightarrow{AB} = \overrightarrow{OC}$

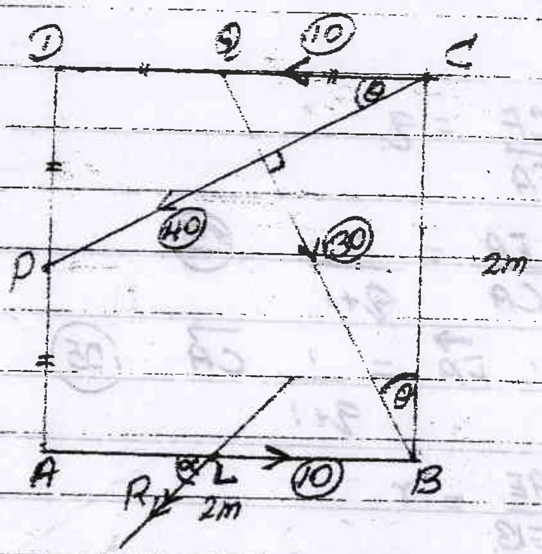
$AB = \parallel OC$

$|\overrightarrow{AB}| = |\overrightarrow{BC}|$

$\therefore$ OABC යනු රොම්බසයකි. (05)

$\boxed{25}$

(08) $\tan\theta = \frac{1}{2}$



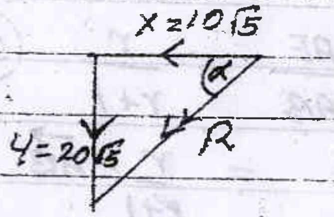
$\therefore \sin\theta = \frac{1}{\sqrt{5}}$, $\cos\theta = \frac{2}{\sqrt{5}}$

$\overrightarrow{AB}$, $X = 10 - 10 + 30\sin\theta - 40\cos\theta$

$= \frac{30}{\sqrt{5}} - \frac{80}{\sqrt{5}}$

$= -10\sqrt{5}$ (05)

$\uparrow AD$, $Y = -40\sin\theta - 30\cos\theta$



$= \frac{-40}{\sqrt{5}} - \frac{60}{\sqrt{5}}$

$= -20\sqrt{5}$ (05)

$R^2 = X^2 + Y^2 = (10\sqrt{5})^2 + (20\sqrt{5})^2 = 2500$

$\underline{R = 50N}$ (05)

$\tan\alpha = \frac{20\sqrt{5}}{10\sqrt{5}} = 2$ $\therefore \sin\alpha = \frac{2}{\sqrt{5}}$, $\cos\alpha = \frac{1}{\sqrt{5}}$

$\curvearrowleft B$ ඝූර්ණ ගැනීමෙන්,

$50 \times BL\sin\alpha = 10 \times 2 + 40 \times 2\cos\theta$ (05)

$50 \times \frac{2}{\sqrt{5}} . BL = 20 + 80 \times \frac{2}{\sqrt{5}}$

$BL = \frac{8 + \sqrt{5}}{5}$

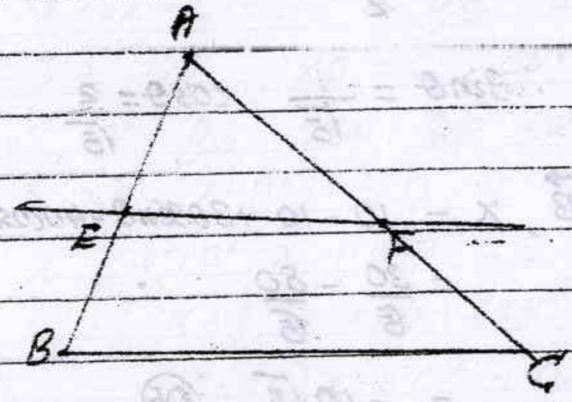
$\therefore AL = 2 - \left(\frac{8 + \sqrt{5}}{5}\right)$

$= \frac{2 - \sqrt{5}}{5}$

$\therefore$ දික් කළ BA මත A සිට $\left(\frac{\sqrt{5} - 2}{5}\right)$m දුරින් L පිහිටයි. (05)

$\boxed{25}$

9) $\frac{CF}{FA} = q$



$\therefore \frac{FA}{CA} = \frac{1}{q+1}$ (05)

$\therefore \overrightarrow{FA} = \frac{1}{q+1} \overrightarrow{CA}$ (05)

$\frac{AE}{EB} = r$

$\therefore \frac{AE}{AB} = \frac{r}{r+1}$ (05)

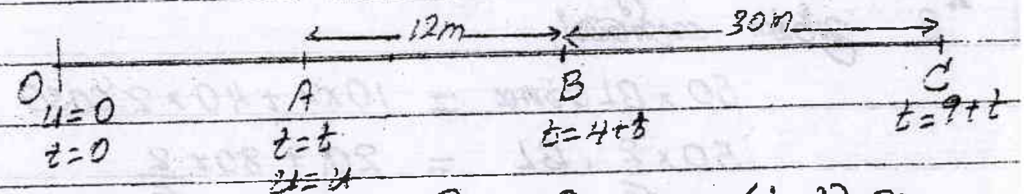
$\overrightarrow{AE} = \frac{r}{r+1} \overrightarrow{AB}$ (05)

$\overrightarrow{EF} = \overrightarrow{EA} + \overrightarrow{AF}$

$\therefore \overrightarrow{EF} = \frac{r}{r+1} \overrightarrow{BA} + \frac{1}{q+1} \overrightarrow{AC}$ වේ. (05)

$\boxed{25}$

10)



(i) දුම්රියේ ඒකාකාර ත්වරණය a ද, දී ඇති කාලය $(t=t)$ විට, දුම්රියේ ප්‍රවේගය u ලෙස ද ගනිමු.

$S = ut + \frac{1}{2}at^2$ යෙදීමෙන්,

$A \rightarrow B$ චලිතයට $\longrightarrow$ $12 = 4u + \frac{1}{2} \times 16a$ (05)

$u + 2a = 3$ ——— ① (05)

$A \rightarrow C$ චලිතයට $\longrightarrow$ $42 = 9u + \frac{1}{2} \times 81a$

$6u + 27a = 28$ ——— ② (05)

② − ① × 6 $\quad a = \frac{2}{3}\ ms^{-2}$ (05)

$\therefore$ ① න් $u = 3 - 2 \times \frac{2}{3}$

$u = \frac{5}{3}\ ms^{-1}$ (05)

$\boxed{25}$

:)

$x^2+bx+c=0$ හි මූල α හා β නම්,

$\alpha+\beta=-b$, $\alpha\beta=c$ වේ. (05)

α^2, β^2 මූල ලෙස ඇති සමීකරණය,

$(x-\alpha^2)(x-\beta^2)=0$ (05)

$x^2-(\alpha^2+\beta^2)x+\alpha^2\beta^2=0$

$x^2-[(\alpha+\beta)^2-2\alpha\beta]x+(\alpha\beta)^2=0$ (05)

$$\underline{\underline{x^2-(b^2-2c)x+c^2=0}} \quad \text{——} ① \quad (05) \qquad \boxed{25}$$

$\lambda=\frac{1}{\alpha^2}+\beta^2$ හා $\mu=\frac{1}{\beta^2}+\alpha^2$ ලෙස ගනිමු.

$\lambda=\frac{1+(\alpha\beta)^2}{\alpha^2}$ (05), $\mu=\frac{1+(\alpha\beta)^2}{\beta^2}$ (05)

$\lambda=\frac{1+c^2}{\alpha^2}$ (05) , $\mu=\frac{1+c^2}{\beta^2}$ (05)

① හි මූල α^2, β^2 නිසා [?] සමීකරණය $(y-\lambda)(y-\mu)=0$ (05)

ආකාරයේ විට, $y=\frac{1+c^2}{x}$ (05) $\therefore$ $x=\frac{1+c^2}{y}$ (05)

$\therefore$ ① න්, $\left(\frac{1+c^2}{y}\right)^2-(b^2-2c)\left(\frac{1+c^2}{y}\right)+c^2=0$ (05)

$$\underline{\underline{c^2y^2-(b^2-2c)(1+c^2)y+(1+c^2)^2=0}} \ (05) \ (c\neq 0) \ \boxed{4:}$$

) $F(x)$, $(x-a)(x-b)(x-c)$ වලින් බෙදුවිට ලබ්ධිය $Q(x)$ ලෙස ගත්විට, ඉතිරිය දෙවන මාත්‍රයේ නිසා එහි ආකාරය $A(x-b)(x-c)+B(x-c)(x-a)+C(x-a)(x-b)$ ලෙස ලිවිය හැකිය. (10)

බෙදීම් ඇල්ගොරිතමයෙන්,

$$F(x) \equiv Q(x)(x-a)(x-b)(x-c)+A(x-b)(x-c)+B(x-c)(x-a)+C(x-a)(x-b) \text{ වේ.} \ (05)$$

$x=a$ විට, $F(a)=A(a-b)(a-c)$ (05)

$$\therefore A=\frac{F(a)}{(a-b)(a-c)}=\frac{-F(a)}{(a-b)(c-a)} \ (05)$$

$x = b$ විට, $F(b) = B(b-c)(b-a)$ 05

$$\therefore B = \frac{F(b)}{(b-c)(b-a)} = \frac{-F(b)}{(b-c)(a-b)}$$ 05

$x = c$ විට, $F(c) = C(c-a)(c-b)$ 05

$$\therefore C = \frac{F(c)}{(c-a)(c-b)} = \frac{-F(c)}{(c-a)(b-c)}$$ 05

$$\therefore \text{ශේෂය} = -\left[\frac{F(a)(x-b)(x-c)}{(a-b)(c-a)} + \frac{F(b)(x-a)(x-c)}{(b-c)(a-b)} + \frac{F(c)(x-a)(x-b)}{(c-a)(b-c)}\right]$$

$g(x) = x^3 - kx^2 + kx + 1$; $k \in \mathbb{R}$

$g(x)$, $(x+1)(x-0)(x-1)$ න් බෙදූ විට ශේෂය Rx නම්,

$a = -1,\ b = 0,\ c = 1$ ලෙසින්, 05

$$Rx = -\left[\frac{g(-1)(x)(x-1)}{(-1-0)(1+1)} + \frac{g(0)(x+1)(x-1)}{(0-1)(-1-0)} + \frac{g(1)(x+1)(x)}{(1+1)(0-1)}\right]$$

$g(-1) = -2k$, $g(0) = 1$, $g(1) = 2$ 05

$$\therefore Rx = -\frac{2k\,x(x-1)}{2} - (x+1)(x-1) + \frac{2(x+1)x}{2}$$ 05

$$= -kx(x-1) - (x+1)(x-1) + x(x+1)$$ 05

ශේෂයේ x^2 පදයේ සංගුණකය ශූන්‍ය වීමට,

$$-k - 1 + 1 = 0$$

$$\therefore k = 0$$ 05

30

15

12)

(a) $x^7 + 1 > x^6 + x$

$(x^7 - x^6) - (x-1) \geq x^6(x-1) - (x-1)$ 05

$= (x-1)(x^6 - 1)$

$= (x-1)\left[(x^2)^3 - 1^3\right]$ 05

$= (x-1)(x^2-1)(x^4+x^2+1)$

$= (x+1)(x-1)^2(x^4+x^2+1)$ 05

$|x| > 1$ නිසා, $(x+1)$, $(x-1)^2$, (x^4+x^2+1) යන විචල්‍යයන් ධන රාශි වේ. $\therefore (x+1)(x-1)^2(x^4+x^2+1) > 0$ වේ. 10

$\therefore x^7 - x^6 - (x-1) > 0$ $\therefore x^7 + 1 > x^6 + x$ 05

3

(12)

(b) $f(x) = (\lambda+2)x^2 - 2(\lambda-2)x + 2$

$$= (\lambda+2)\left[x^2 - \frac{2(\lambda-2)}{(\lambda+2)}x + \frac{2}{\lambda+2}\right] \quad (05)$$

$$= (\lambda+2)\left[x - \left(\frac{\lambda-2}{\lambda+2}\right)\right]^2 + 2 - \frac{(\lambda-2)^2}{\lambda+2}$$

$$= (\lambda+2)\left[x - \left(\frac{\lambda-2}{\lambda+2}\right)\right]^2 + \frac{2(\lambda+2)-(\lambda-2)^2}{\lambda+2} \quad ① \ (05)$$

(i) x හි සියලු තාත්වික අගයන් සඳහා $f(x) > 0$ වීම සඳහා (05)

$\lambda+2 > 0$ සහ $\dfrac{2(\lambda+2)-(\lambda-2)^2}{\lambda+2} > 0$ (05)

$$2(\lambda+2) - (\lambda-2)^2 > 0$$

$$\lambda^2 - 6\lambda < 0$$

$$\lambda(\lambda-6) < 0 \quad (05)$$

$\therefore \ 0 < \lambda < 6$ වේ. (05)

$\therefore$ λ ට ගත හැකි විශාලතම නිඛිලමය අගය 5 (05) $\boxed{35}$

විකල්ප ක්‍රමයක්. x හි සියලු තාත්වික අගයන් සඳහා

$f(x) > 0$ වීමට $\Delta < 0$ විය යුතුයි (10)

එනම්, $4(\lambda-2)^2 - 4(\lambda+2)\times 2 < 0$ (10)

$$\lambda^2 - 4\lambda + 4 - 2\lambda - 4 < 0$$

$$\lambda^2 - 6\lambda < 0$$

$$\lambda(\lambda-6) < 0 \quad (05)$$

$\therefore \ 0 < \lambda < 6$ (05)

$\therefore$ λ ට ගත හැකි විශාලතම නිඛිලමය අගය $= 5$ (05)

$\boxed{35}$

(ii) $f(x) = 0$ සමීකරණයට හරියටම තාත්වික එක් මූලයක් තිබීමට නම්,

① න්, $\dfrac{2(\lambda+2)-(\lambda-2)^2}{\lambda+2} = 0$ වේ. (05)

$$\lambda(\lambda-6) = 0 \quad (05)$$

$\therefore$ $\lambda = 0$ (05) හෝ $\lambda = 6$ (05) නම්, $f(x)=0$ සමීකරණයට

හරියටම එක් තාත්වික මූලයක් පමණක් තිබේ. (05)

$\boxed{25}$

විකල්ප ක්‍රමයක්

$f(x) = 0$ සමීකරණයට සමපාත තාත්වික මූල දෙකක් තිබීමට නම් $\Delta x = 0$ විය යුතුයි. 10

එනම් $4(\lambda-2)^2 - 4(\lambda+2) \times 2 = 0$

$\lambda(\lambda-6) = 0$ 05

05 $\lambda = 0$ හෝ $\lambda = 6$ 05 $\boxed{25}$

(iii) $\lambda = 1$ විට, ① න්

$$f(x) = 3\left(x+\frac{1}{3}\right)^2 + \frac{5}{3} \quad 05$$

$\therefore f(x)$ හි අවම අගය $\frac{5}{3}$ වන්නේ $x = -\frac{1}{3}$ විට වේ. 05

$$\frac{1}{f(x)} = \frac{1}{3\left(x+\frac{1}{3}\right)^2 + \frac{5}{3}} \quad 05$$

$x = -\frac{1}{3}$ විට $f(x)$ හි උපරිම අගය $\frac{3}{5}$ වේ. 05

$\boxed{20}$

(C) $f : \mathbb{R} \longrightarrow \mathbb{R}, \quad x \neq 1$

$$f(x) = \frac{x}{x-1} \Rightarrow y = \frac{x}{x-1}$$

$$x(y-1) = y$$

$$x = \frac{y}{y-1}, \quad y \neq 1 \quad 05$$

(i) f හි පරාසය $y = \{ y : y \in \mathbb{R},\ y \neq 1 \}$ 10 $\boxed{15}$

(ii)

$$f \circ f(x) = \frac{x/(x-1)}{\left(\frac{x}{x-1} - 1\right)} \quad 05$$

$$f \circ f(x) = \frac{x}{x-x+1} = x \qquad \therefore f \circ f(x) = x \quad 10$$

$f^{-1}(x) \longrightarrow \frac{x}{x-1}$ වේ. (f ශ්‍රිතයේ ප්‍රතිලෝමය f ම 10

$\boxed{2}$

$\boxed{150}$

(ii)

$3\sin 2x + 2(\sin x - \cos x) + 2 = 0$

$-3(\sin^2 x - 2\sin x\cos x + \cos^2 x) + 2(\sin x - \cos x) + 5 = 0$

$-3(\sin x - \cos x)^2 + 2(\sin x - \cos x) + 5 = 0$ (05)

$-3t^2 + 2t + 5 = 0$; ($\sin x - \cos x = t$ ලෙස ගනිමු)

$3t^2 - 2t - 5 = 0$

$(3t - 5)(t + 1) = 0$ (05)

$3t - 5 = 0$ හෝ $t + 1 = 0$

$t = \frac{5}{3}$ (05) හෝ $t = -1$ (05)

$t = \frac{5}{3}$ විට, $\sin x - \cos x = \frac{5}{3}$ (05)

$\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x\right) = \frac{5}{3}$

$\sin x \sin \frac{\pi}{4} - \cos x \cos \frac{\pi}{4} = \frac{5}{3\sqrt{2}}$

$\sin(x - \frac{\pi}{4}) = \frac{5}{3\sqrt{2}}$ (05)

$\sin(x - \frac{\pi}{4}) = \sin\beta$ $\left(\beta = \sin^{-1}\frac{5}{3\sqrt{2}}\right)$

$x - \frac{\pi}{4} = n\pi + (-1)^n\beta$ (05)

$x = n\pi + (-1)^n\beta + \frac{\pi}{4}$, $n \in \mathbb{Z}$, (05)

$\beta = \sin^{-1}\frac{5}{3\sqrt{2}}$

$t = -1$ විට, $\sin x - \cos x = -1$ (05)

$\cos x - \sin x = 1$

$\cos(x + \frac{\pi}{4}) = \cos\frac{\pi}{4}$

$x + \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$ (05)

$x = 2n\pi$ (05) හෝ $2n\pi - \frac{\pi}{2}$ (05) , $n \in \mathbb{Z}$ 60

විකල්ප ක්‍රමයක්

$\sin x - \cos x = -1$ (05)

$\sin(x - \frac{\pi}{4}) = \sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \sin(-\frac{\pi}{4})$ (05)

$x - \frac{\pi}{4} = n\pi + (-1)^n(-\frac{\pi}{4})$

$x = n\pi - (-1)^n\frac{\pi}{4} + \frac{\pi}{4}$, $n \in \mathbb{Z}$

(10)

(13)

(b) $6\cot 2\theta + 5\cot\theta - 5\tan\theta = 0$

$$6\frac{(1-\tan^2\theta)}{2\tan\theta} + \frac{5}{\tan\theta} - 5\tan\theta = 0 \quad (10)$$

$$\frac{6(1-t^2)}{2t} + \frac{5}{t} - 5t = 0 \quad (\tan\theta = t \text{ ලෙස ගනිමු.})$$

$$3(1-t^2) + 5 - 5t^2 = 0 \quad (10)$$

$$8t^2 = 8$$

$$t = \pm 1 \quad (10)$$

$$\tan\theta = \pm 1$$

$$\tan\theta = \tan(\pm\pi/4) \quad (05)$$

$$\underline{\underline{\theta = n\pi \pm \pi/4 \,,\; n\in\mathbb{Z}}} \quad (10)$$

$\boxed{45}$

(c) $\sin 3\theta - \sin\theta = \cos 3\theta - \cos\theta$

$$\sin 3\theta - \sin\theta + \cos\theta - \cos 3\theta = 0$$

$$2\cos 2\theta \sin\theta + 2\sin 2\theta \sin\theta = 0 \quad (10)$$

$$2\sin\theta(\cos 2\theta + \sin 2\theta) = 0$$

$\sin\theta = 0$ (05) හෝ $\cos 2\theta + \sin 2\theta = 0$ (05)

$\sin\theta = \sin 0$ $\qquad$ $\sin 2\theta = -\cos 2\theta$

$\underline{\underline{\theta = n\pi,\; n\in\mathbb{Z}}}$ (05) $\qquad$ $\tan 2\theta = -1$ (05)

$$\tan 2\theta = \tan(-\pi/4) \quad (05)$$

$$2\theta = n\pi - \pi/4 \quad (05)$$

$$\theta = \frac{n\pi}{2} - \frac{\pi}{8} \text{ ලෙස ල}$$

$$\underline{\theta = \frac{\pi}{8}(4n-1),\; n\in}$$

විකල්ප ක්‍රමයක්:

$$\cos 2\theta + \sin 2\theta = 0$$

$$\cos\left(2\theta - \frac{3\pi}{4}\right) = \cos 0 \quad (10)$$

$$2\theta - \pi/4 = 2n\pi \quad (05)$$

$$\theta = n\pi + \pi/8,\; n\in\mathbb{Z} \quad (10)$$

$\boxed{15}$

14)

(a) $\sin^{-1} 3/5 + \tan^{-1} 3/5 = \pi/4 + \tan^{-1} 8/19$

$\sin^{-1} 3/5 = \alpha \Rightarrow \sin\alpha = 3/5 \quad \therefore \tan\alpha = 3/4 \quad (\because 0 < \alpha < \pi/2)$

$\tan^{-1} 3/5 = \beta \Rightarrow \tan\beta = 3/5$

L.H.S $= y$ නම්, $y = \alpha + \beta$

$$\tan y = \tan(\alpha+\beta)$$

$$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \quad (05)$$

$$= \frac{3/4 + 3/5}{1 - 3/4 \cdot 3/5} \quad (05) = \frac{27}{11} \quad (05)$$

$$= \frac{1 + 8/19}{1 - 8/19} \quad (05)$$

$$= \frac{\tan\pi/4 + \tan\theta}{1 - \tan\pi/4 \tan\theta} \quad (05) \quad (\tan\theta = 8/19)$$

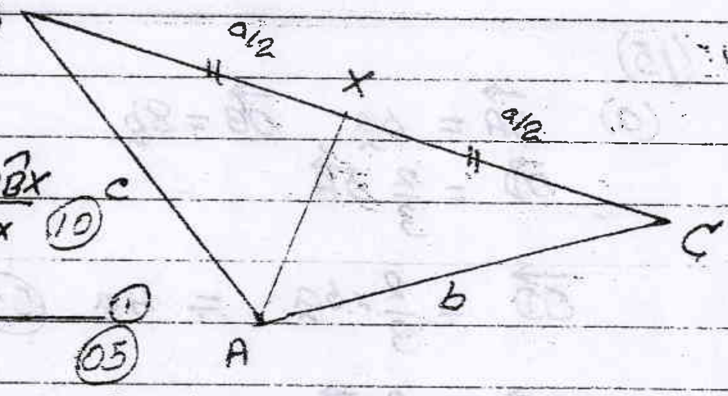
$$= \tan(\pi/4 + \theta) \quad (05)$$

$$\therefore \alpha + \beta = \pi/4 + \theta \quad (05)$$

$$\underline{\underline{\sin^{-1} 3/5 + \tan^{-1} 3/5 = \pi/4 + \tan^{-1} 8/19}} \quad (05)$$

$\boxed{40}$

b)



ABX ත්‍රිකෝණයට sin නීතිය යෙදීමෙන්,

$$\frac{\sin B\hat{A}X}{BX} = \frac{\sin A\hat{X}B}{AB} = \frac{\sin A\hat{B}X}{AX} \quad (10)$$

$$\therefore \sin A\hat{X}B = c\frac{\sin B}{AX} \quad \text{——} \ (1) \quad (05)$$

ABX ත්‍රිකෝණයට cosin නීතිය යෙදීමෙන්,

$$AB^2 = BX^2 + AX^2 - 2AX \cdot BX\cos A\hat{X}B \quad \text{——} \ (2) \quad (10)$$

ACX ත්‍රිකෝණයට cosin නීතිය යෙදීමෙන්,

$$AC^2 = CX^2 + AX^2 - 2AX\,CX\cos(\pi - A\hat{X}B) \quad \text{——} \ (3) \quad (10)$$

$BX = XC \quad \therefore$ (2) + (3) න්, $AB^2 + AC^2 = 2BX^2 + 2AX^2$ (05)

$c^2 + b^2 = 2(a/2)^2 + 2AX^2$ (05)

$\therefore AX^2 = \dfrac{2(b^2+c^2) - a^2}{4}$ (05)

$\therefore AX = \dfrac{\sqrt{2b^2+2c^2-a^2}}{2}$ (05)

$\therefore$ ① න් $\sin A\hat{X}B = \dfrac{2c\sin B}{\sqrt{2b^2+2c^2-a^2}}$ (05)

$\sin A\hat{X}B = \dfrac{2b\sin C}{\sqrt{2b^2+2c^2-a^2}}$ (05) (sin නීතියෙන්. $c\sin B = b\sin$ (05)

(c) $4\cos^2 x - 24\sin x\cos x - 6\sin^2 x = 4$

$2(1+\cos 2x) - 12\sin 2x + 3(\cos 2x - 1) = 4$ (05)

$5\cos 2x - 12\sin 2x = 5$ (05)

$\dfrac{5}{13}\cos 2x - \dfrac{12}{13}\sin 2x = \dfrac{5}{13}$ (05)

$\cos 2x\cos\beta - \sin 2x\sin\beta = \dfrac{5}{13}$ $(\cos\beta = 5/13, \sin\beta =$

$\cos(2x+\beta) = \cos\beta$ (05)

මෙය $a\cos(2x+\beta) = \cos\beta$ ආකාරයේ වේ.

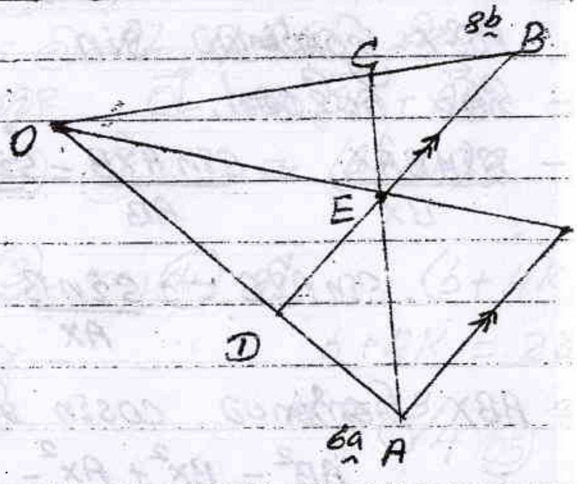
මෙහි $a = 1$ (05), $\beta = \cos^{-1} 5/13$ (05)

(15)

(a) $\overrightarrow{OA} = 6\underset{\sim}{a}$ $\overrightarrow{OB} = 8\underset{\sim}{b}$

$\overrightarrow{OD} = \dfrac{2}{3}\overrightarrow{OA}$

$\overrightarrow{OD} = \dfrac{2}{3} \times 6\underset{\sim}{a} = 4\underset{\sim}{a}$ (05)



$\overrightarrow{OC} = \dfrac{3}{4}\overrightarrow{OB}$

$= \dfrac{3}{4} \times 8\underset{\sim}{b} = 6\underset{\sim}{b}$ (05)

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$\overrightarrow{AC} = 6(\underset{\sim}{b} - \underset{\sim}{a})$ (05)

(15)

(a) $\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OD}$

$= 4\underline{a} - 8\underline{b}$ (05)

$\overrightarrow{BD} = 4(\underline{a} - 2\underline{b})$

20

$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE} = 6\underline{a} + \lambda\overrightarrow{AC} = 6\underline{a} + \lambda \cdot 6(\underline{b} - \underline{a})$

$\overrightarrow{OE} = 6(1-\lambda)\underline{a} + 6\lambda\underline{b}$ ——— ① (05)

තවද, $\overrightarrow{OE} = \overrightarrow{OB} + \overrightarrow{BE}$

$= 8\underline{b} + \mu\overrightarrow{BD} = 8\underline{b} + 4\mu(\underline{a} - 2\underline{b})$

$\overrightarrow{OE} = 4\mu\underline{a} + 8(1-\mu)\underline{b}$ ——— ② (05)

① = ② ත්,

$6(1-\lambda)\underline{a} + 6\lambda\underline{b} = 4\mu\underline{a} + 8(1-\mu)\underline{b}$

$6(1-\lambda) = 4\mu \quad , \quad 6\lambda = 8(1-\mu)$

$3 - 3\lambda = 2\mu \qquad 3\lambda = 4 - 4\mu$

$\lambda = \frac{2}{3}$ (05) හා $\mu = \frac{1}{2}$ (05) වේ.

$\therefore \overrightarrow{OE} = 2\underline{a} + 4\underline{b} \quad (2(\underline{a} + 2\underline{b}))$ (05)

25

O, E හා L ලක්ෂ්‍ය ඒක රේඛීය බැවින්,

$\overrightarrow{OL} = \gamma\overrightarrow{OE}$

$= \gamma(2\underline{a} + 4\underline{b})$ ——— ③ (05)

AL // DB නිසා, $\overrightarrow{AL} = K\overrightarrow{DB}$ ලෙස ලිවිය හැකිය.

තවද, $\overrightarrow{OL} = \overrightarrow{OA} + \overrightarrow{AL} = 6\underline{a} + 4K(\underline{a} - 2\underline{b})$

$= (6+4K)\underline{a} - 8K\underline{b}$ ——— ④ (05)

③ හා ④ ත්, $(6+4K)\underline{a} - 8K\underline{b} = \gamma(2\underline{a} + 4\underline{b})$

$6 + 4K = 2\gamma \quad , \quad -8K = 4\gamma$

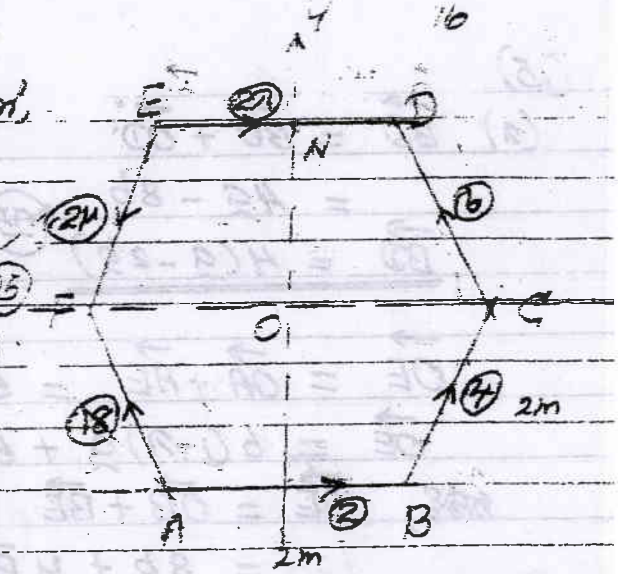
$K = -\frac{3}{4}$ (05) හා $\gamma = \frac{3}{2}$ (05) වේ.

$\therefore \overrightarrow{OL} = 3(\underline{a} + 2\underline{b})$ (05)

25

(5) (b) බල පද්ධතිය සමතුලිතතාවේ පවතින්,

$\vec{OC}$ ඔස්සේ විභේදනයෙන්,



$2+2\lambda+4\cos 60^\circ-6\cos 60^\circ$

$-2\mu\cos 60^\circ-18\cos 60^\circ=0$ (05)

$2\lambda-\mu=8$ —— ① (05)

$\vec{ON}$ ඔස්සේ විභේදනයෙන්,

$(4+6+18-2\mu)\sin 60^\circ=0$ (05)

$\underline{\mu=14}$ (05)

① න් $\underline{\lambda=11}$ (05)

[25]

$\overset{\curvearrowleft}{O}$ වටා ඝූර්ණ ගැනීමෙන්,

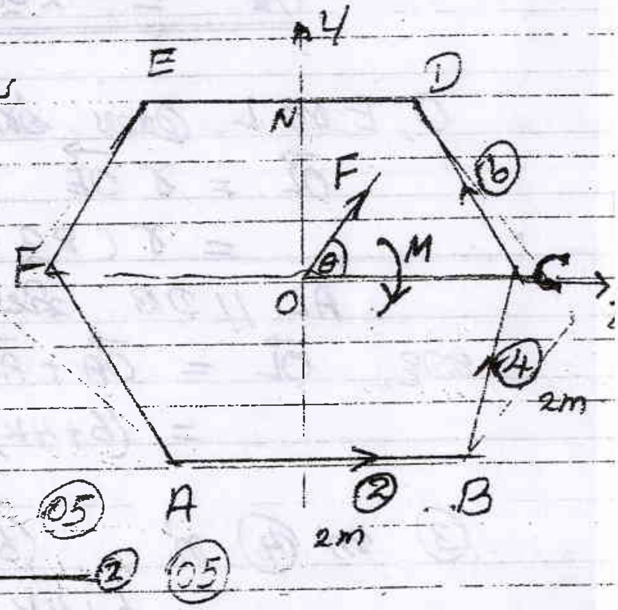
$G=(2+4+6-2\lambda+2\mu-18)\times 2\sin 60^\circ$ (05)

$=(-6-2\times 11+2\times 14)\,2\times\frac{\sqrt{3}}{2}$

$G=0$ (05)

λ හා μ ට ඉහත අගයන් යෙදූ විට, G=0 න් බල පද්ධතිය සමතුලිතතාවේ පිහිටි බව තවදුරටත් සනාථ වේ. (05)

O හි යොදන ඒකතල බලයේ විශාලත්වය F හා එය OC සමග θ කෝණයක් සාදන්නේ යැයි ද, එකතු කරන බල යුග්මය M හා එය දක්ෂිණාවර්තව ක්‍රියා කරන්නේ යැයි ද සිතමු.



(ii) බල පද්ධතිය සමතුලිතතාවේ පවතින්,

$\vec{OC}$ ඔස්සේ විභේදනයෙන්

$F\cos\theta+2+(4-6)\cos 60^\circ=0$ (05)

$F\cos\theta=-1$ —— ② (05)

$\vec{ON}$ ඔස්සේ විභේදනයෙන්

$F\sin\theta+(6+4)\sin 60^\circ=0$ (05)

$F\sin\theta=-5\sqrt{3}$ —— ③ (05)

$③^2+②^2 \qquad F^2=1+75$

$=76$

$\therefore\ \underline{F=2\sqrt{19}\,N}$ (05)

හෝ $\sqrt{76}\,N$

[25]

7 (ii) O ඝූර්ණ ගැනීමෙන්,

$$(2+4+6)\times 2\sin 60^\circ - M = 0 \quad (10)$$

$$\therefore M = 12\sqrt{3}$$

යොදන යුතු බල යුග්මයේ විශාලත්වය $12\sqrt{3}$ N (05) $\boxed{15}$

$\boxed{150}$

$O \equiv (0,0)$, $A \equiv (1,3)$, $B \equiv (2,-1)$ $C \equiv (2,4)$

$\therefore \overrightarrow{OA} = \underset{\sim}{i} + 3\underset{\sim}{j}$ (05), $\overrightarrow{OB} = 2\underset{\sim}{i} - \underset{\sim}{j}$ (05), $\overrightarrow{OC} = 2\underset{\sim}{i} + 4\underset{\sim}{j}$ (05)

$\underline{\underset{\sim}{F}_1 = \overrightarrow{AO} = -\underset{\sim}{i} - 3\underset{\sim}{j}}$ (05)

$\underset{\sim}{F}_2 = 2\overrightarrow{OB} = 2(2\underset{\sim}{i} - \underset{\sim}{j})$

$\underline{\underset{\sim}{F}_2 = 4\underset{\sim}{i} - 2\underset{\sim}{j}}$ (05)

$\underset{\sim}{F}_3 = 3\overrightarrow{BC} = 3(\overrightarrow{BO} + \overrightarrow{OC}) = 3(-2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{i} + 4\underset{\sim}{j})$

$\underline{\underset{\sim}{F}_3 = 15\underset{\sim}{j}}$ (05)

$\underset{\sim}{F}_4 = 4\overrightarrow{AC} = 4(\overrightarrow{AO} + \overrightarrow{OC}) = 4(-\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{i} + 4\underset{\sim}{j})$

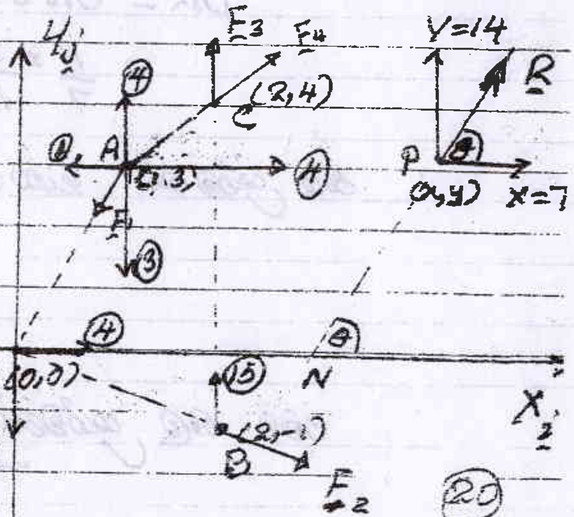
$\underline{\underset{\sim}{F}_4 = 4\underset{\sim}{i} + 4\underset{\sim}{j}}$ (05) $\boxed{35}$

සම්ප්‍රයුක්තය $\underset{\sim}{R}$ නම්

$\underset{\sim}{R} = \underset{\sim}{F}_1 + \underset{\sim}{F}_2 + \underset{\sim}{F}_3 + \underset{\sim}{F}_4$ වේ.

$= (-\underset{\sim}{i} - 3\underset{\sim}{j}) + (4\underset{\sim}{i} - 2\underset{\sim}{j}) + (15\underset{\sim}{j}) + (4\underset{\sim}{i} + 4\underset{\sim}{j})$ (05)

$\underset{\sim}{R} = 7\underset{\sim}{i} + 14\underset{\sim}{j}$ (05)



$|\underset{\sim}{R}| = \sqrt{49+196} = \sqrt{245}$

$\underline{|\underset{\sim}{R}| = 7\sqrt{5}}$ (05)

සම්ප්‍රයුක්ත බලය Ox සමඟ θ කෝණයක් සාදයි නම්,

$$\tan\theta = \frac{14}{7} \;(05) = 2$$

$$\underline{\theta = \tan^{-1} 2} \quad (05)$$

$\boxed{45}$

සම්ප්‍රයුක්ත බලය $F \equiv (x, y)$ හි දී ක්‍රියා කරයි නම්,

$\circlearrowleft O$ ඝූර්ණ ගැනීමෙන්,

$$14x - 7y = (4-3)\times 1 + (1-4)\times 3 + 15\times 2 \quad (10)$$

$$14x - 7y = 22$$

$\therefore$ සම්ප්‍රයුක්තයේ ක්‍රියා රේඛාවේ සමීකරණය $\Rightarrow$

$$\underline{14x - 7y = 22.} \quad (10) \qquad \boxed{20}$$

$\underset{\sim}{F}$ බලයක් එකතු කිරීම නිසා, නව බල පද්ධතිය, බල යුග්මයකට තුල්‍ය වේ නම්, $\underset{\sim}{F} + \underset{\sim}{R} = \underset{\sim}{0}$ වේ. (10)

එවිට, $\underset{\sim}{F} = -\underset{\sim}{R}$

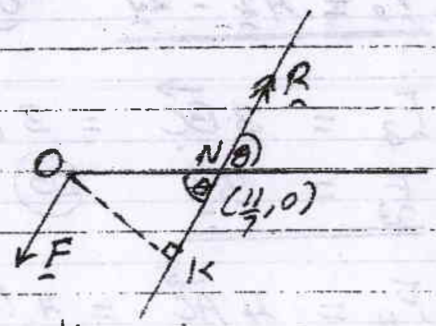
$$\underline{\underset{\sim}{F} = -7\underset{\sim}{i} - 14\underset{\sim}{j}} \quad (05) \qquad \boxed{15}$$

$\underset{\sim}{R}$ හි ක්‍රියා රේඛාව x අක්ෂය හමුවන ලක්ෂ්‍යය N නම්,

$$14x = 22$$

$$x = \frac{11}{7} \quad (05)$$

$$\therefore N \equiv \left(\frac{11}{7}, 0\right) \quad (05)$$



$\therefore$ O සිට $\underset{\sim}{R}$ හි ක්‍රියා රේඛාවට ලම්බක දුර $= OK$

$$OK = ON \sin\theta \quad (05)$$

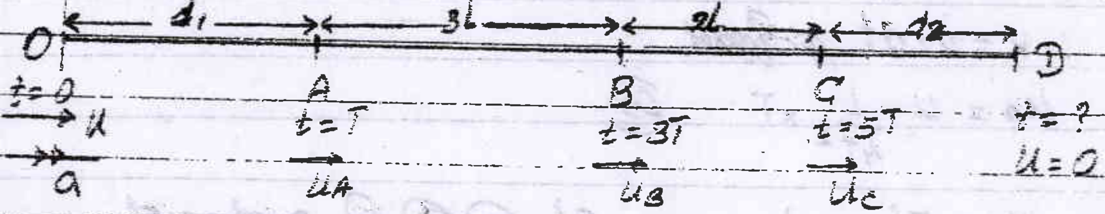
$$= \frac{11}{7} \times \frac{2}{\sqrt{5}} = \frac{22}{7\sqrt{5}} \quad (05)$$

$\therefore$ බල යුග්මයේ ඝූර්ණය $= 7\sqrt{5} \times \frac{22}{7\sqrt{5}}$ (05)

$$\underline{= 22 \text{ Nm}} \quad (05)$$

මෙම බල යුග්මයේ $\underline{\text{දිශාව වාමාවර්ත වේ.}}$ (05) $\boxed{35}$

$\boxed{150}$



අංශුවේ ඒකාකාර ත්වරණය a ලෙසත්, O හා A හිදී ප්‍රවේග පිළිවෙලින් u හා u_A ලෙස ගනිමු.
$OA = d_1$, $CD = d_2$ ද ලෙස ගනිමු.

— B චලිතයට $\longrightarrow$ $s = ut + \frac{1}{2}at^2$ යෙදීමෙන්,

$$3l = u_A \times 2T + \frac{1}{2}a(2T)^2$$

$$3l = 2u_AT + 2aT^2 \quad \text{①} \quad 10$$

— C චලිතයට $\longrightarrow$ $s = ut + \frac{1}{2}at^2$ යෙදීමෙන්,

$$5l = u_A \times 4T + \frac{1}{2}a(4T)^2$$

$$5l = 4u_AT + 8aT^2 \quad \text{②} \quad 10$$

② − ①×2 $\quad -l = 4aT^2$

$$\therefore a = \frac{-l}{4T^2} \quad 05$$

① න්, $\quad 3l = 2u_AT + 2\left(\frac{-l}{4T^2}\right)T^2$

$$u_A = \frac{7l}{4T} \quad 05$$

— D චලිතයට $\longrightarrow$ $v^2 = u^2 + 2as$ යෙදීමෙන්,

$$0 = \left(\frac{7l}{4T}\right)^2 - 2 \times \frac{l}{4T^2}(5l + d_2) \quad 10$$

$$d_2 = \frac{9l}{8}$$

$$\therefore CD \text{ දුර} = \frac{9l}{8} \quad 05$$

$\boxed{45}$

— A චලිතයට $\longrightarrow$ $v^2 = u^2 + 2as$ යෙදීමෙන්,

$$u_A^2 = u^2 + 2ad_1$$

$$u^2 = \left(\frac{7l}{4T}\right)^2 - 2\left(\frac{-l}{4T^2}\right)d_1 \quad 05$$

$u^2 = \frac{l}{16T^2}(49l + 8d_1)$ ———— (3) (5)

$\longrightarrow$ $v = u + at$ යෙදීමෙන්,

$u_A = u - \frac{l}{4T^2} \times T$ (05)

$u = \frac{7l}{4T} + \frac{l}{4T} = \frac{2l}{T}$ (05) (3) හි ආදේශයෙන්,

$\frac{4l^2}{T^2} = \frac{l}{16T^2}(49l + 8d_1)$ (05)

$64l = 49l + 8d_1$

$d_1 = \frac{15l}{8}$

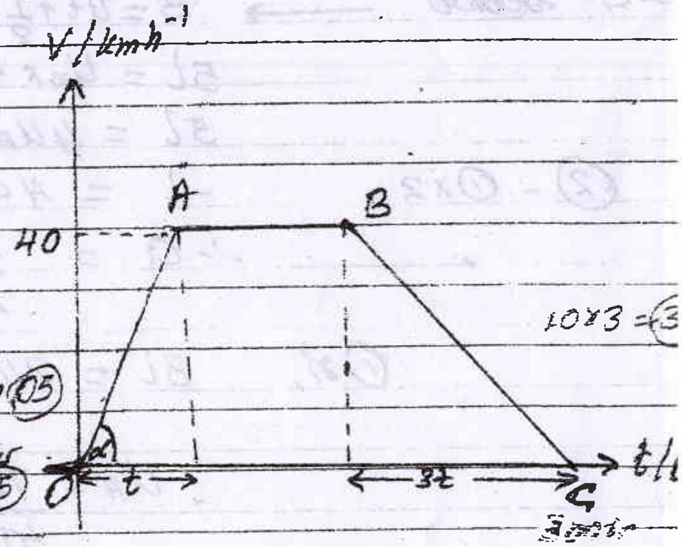
(ii) ∴ OA දුර $= \frac{15l}{8}$ (05) $\boxed{35}$

(b)

ත්වරණයෙන් ගමන් කරන වූ කාලය t නම්, මන්දනයෙන් ගමන් කරන වූ කාලය $3t$ වේ.

∴ නියත ප්‍රවේගයෙන් ගමන් කරන කාලය $= (5 - 4t)$ min (05)

දුම්රිය ගමන් කළ දුර = OABC වර්ගඵලය (05)



$\frac{1}{2} \times 40 \left(\frac{5 + 5 - 4t}{60}\right) = 3$ (10)

$\frac{10 - 4t}{3} = 3$

$4t = 1 \Rightarrow t = \frac{1}{4}$ min (05)

∴ දුම්රිය ත්වරණයෙන් ගිය කාලය $= 15\,S$

∴ දුම්රියේ ත්වරණය $= \tan\alpha = \frac{40\ \text{kmh}^{-1}}{\frac{1}{4}\ \text{min}}$ (10)

$= 160\ \text{km h}^{-1}\text{min}^{-1}$ (05) $\boxed{70}$

$\boxed{150}$