MARKING SCHEME

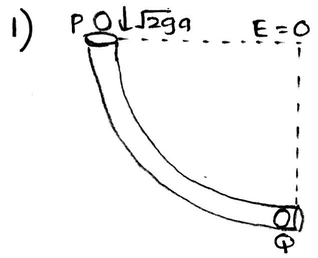
G.C.E (Advance Level) - March 2020

Grade 13 - Term Test II

COMBINED MATHEMATICS II

PART A

1)



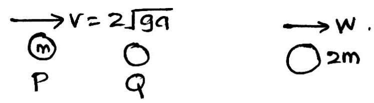
Applying the Law of conservation of energy for P,

$$0 + \frac{1}{2} m.2ga = -mga + \frac{1}{2} mv^2 \quad (10)$$

$$2ga = -2ga + v^2$$

$$v^2 = 4ga$$

$$v = 2\sqrt{ga} \quad (5)$$



Applying the law of Conservation of energy,

$$\rightarrow \quad m.2\sqrt{ga} = 2m\,W \quad (5)$$

$$W = \sqrt{ga} \quad (5)$$

(25)

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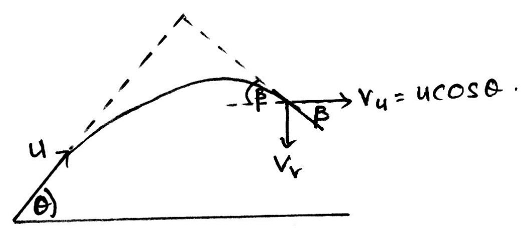
2)



$$\beta + \theta = 90^\circ$$

$$\tan(\beta + \theta) = \tan 90^\circ$$

$$\frac{\tan\beta + \tan\theta}{1 - \tan\beta\tan\theta} \rightarrow \infty$$

$$1 - \tan\beta\tan\theta = 0$$

$$\tan\beta\tan\theta = 1 \quad (5)$$

(1)

$$\rightarrow \quad V_u = u\cos\theta$$

$$\uparrow \quad v = u + at$$

$$V_r = u\sin\theta - gt$$

$$\downarrow \quad -V_r = gt - u\sin\theta \quad (5)$$

$$\tan\beta = \frac{-V_r}{V_u} = \frac{gt - u\sin\theta}{u\cos\theta} \quad (5)$$

from (1),

$$\left(\frac{gt - u\sin\theta}{u\cos\theta}\right)\tan\theta = 1 \quad (5)$$

$$\left(\frac{gt - u\sin\theta}{u\cos\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right) = 1$$

$$gt \sin\theta - u\sin^2\theta = u\cos^2\theta$$

$$gt\sin\theta = u(\sin^2\theta + \cos^2\theta)$$

$$t = \frac{u}{g\sin\theta} \quad (5)$$

$\boxed{25}$

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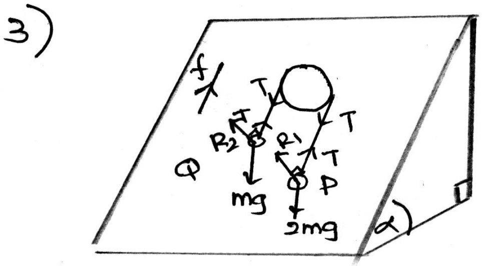
3)



(5) diagram

Applying $F = ma$ for P,

$$2mg\sin\alpha - T = 2mf \quad \text{——} (1) \qquad (5)$$

Applying $F = ma$ for Q

$$T - mg\sin\alpha = mf \quad \text{——} (2) \qquad (5)$$

(1) + (2),
$$mg\sin\alpha = 3mf$$

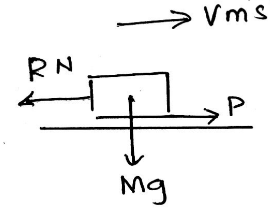
$$f = \frac{g\sin\alpha}{3} \quad (5)$$

From (2),

$$T = mg\sin\alpha + \frac{mg\sin\alpha}{3}$$

$$T = \frac{4mg\sin\alpha}{3} \quad (5)$$

$\boxed{25}$

4) 

For car, $H = Pv$

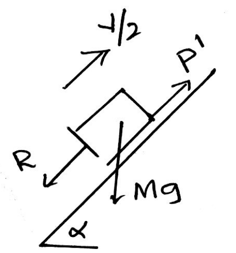
$$H \times 10^3 = Pv$$

$$P = \frac{H \times 10^3}{v} \text{ N} \quad 5$$

Applying $F = ma \rightarrow$

$$P - R = 0$$

$$P = R = \frac{H \times 10^3}{v} \text{ N} \quad 5$$



For the motion on the inclined plane

$$H = Pv$$

$$H \times 10^3 = P' \frac{v}{2}$$

$$P' = \frac{2H \times 10^3}{v} \text{ N}. \quad 5$$

Applying $\nearrow$ $F = ma$

$$P' - R - Mg \sin\alpha = Ma \quad 5$$

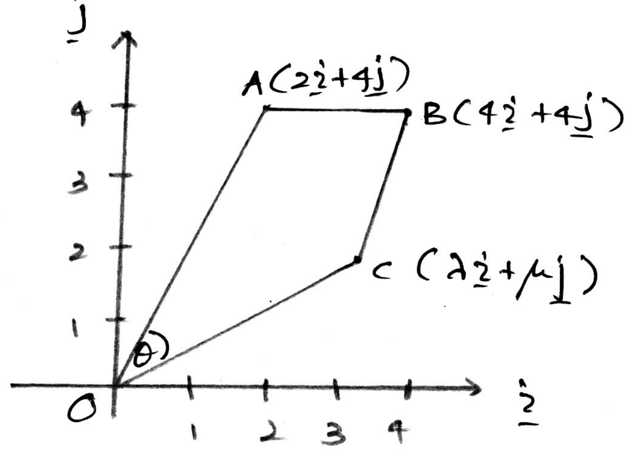
$$\frac{2H \times 10^3}{v} - \frac{H \times 10^3}{v} - Mg \sin\alpha = Ma$$

$$a = \left(\frac{H \times 10^3}{Mv} - g \sin\alpha \right) \text{ m s}^{-2}$$

5

$\boxed{25}$

5)



As $OA \parallel CB$ and $OA = 2CB$

$$\overrightarrow{OA} = 2\overrightarrow{CB} \quad (5)$$

$$2\underline{i}+4\underline{j} = 2\left[4\underline{i}+4\underline{j} - (\lambda\underline{i}+\mu\underline{j})\right] \quad (5)$$

$$2\underline{i}+4\underline{j} = 2\left[(4-\lambda)\underline{i} + (4-\mu)\underline{j}\right]$$

$$(2-8+2\lambda)\underline{i} + [4-2(4-\mu)]\underline{j} = 0$$

$$(-6+2\lambda)\underline{i} + (-4+2\mu)\underline{j} = 0$$

$$-6+2\lambda = 0 \qquad -4+2\mu = 0$$

$$\lambda = 3 \qquad \mu = 2 \quad (5) \text{ both}$$

$$\therefore \overrightarrow{OC} = (3\underline{i}+2\underline{j})$$

$$\overrightarrow{OA}.\overrightarrow{OC} = |\overrightarrow{OA}|.|\overrightarrow{OC}|\cos\theta.$$

$$\overrightarrow{OA}.\overrightarrow{OC} = |\overrightarrow{OA}|.|\overrightarrow{OC}|\cos\theta. \quad (5)$$

$$(2\underline{i}+4\underline{j}).(3\underline{i}+2\underline{j}) = |\overrightarrow{OA}|.|\overrightarrow{OC}|\cos\theta.$$

$$6+8 = \sqrt{20}.\sqrt{13}\cos\theta.$$

$$\cos\theta = \frac{14}{\sqrt{20}\sqrt{13}} = \frac{14}{2\sqrt{5}\sqrt{13}}$$

$$\cos\theta = \frac{7}{\sqrt{65}} \quad (5)$$

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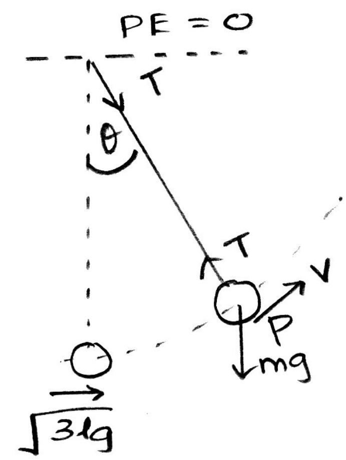
6)



$\cos\theta = \frac{3}{5}$

Applying the Law of Conservation of energy

$$\frac{1}{2}mv^2 - mgl\cos\theta = \frac{1}{2}.m.3lg - mgl. \quad (10)$$

$$v^2 = 3lg - 2lg + 2lg.\frac{3}{5}$$

$$v^2 = \sqrt{\frac{11lg}{5}} \quad (5)$$

For P,

Applying $F = ma$ ↖

$$T - mg\cos\theta = \frac{mv^2}{l} \quad (5)$$

$$T = \frac{11mlg}{5l} + \frac{3mg}{5}$$

$$T = \frac{14mg}{5} \quad (5)$$

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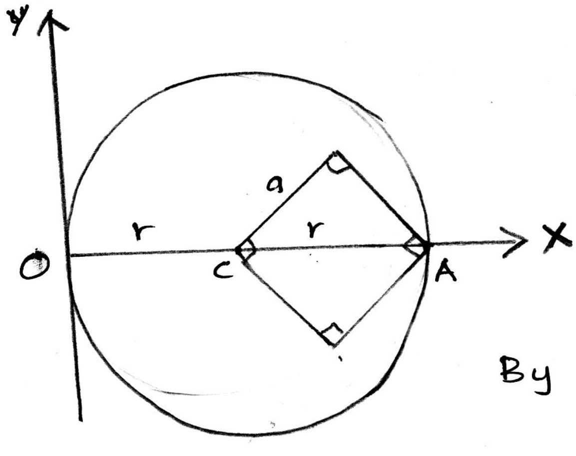
7)



$2a^2 = r^2$

$a = \frac{r}{\sqrt{2}}$

Density $-\rho$

By symmetry

$\bar{Y} = 0.$

Object	Mass	Distance from O to center. g.
◯	$\pi r^2 \rho$	r
◇	$\frac{r^2\rho}{2}$	$\frac{3r}{2}$
Composite body	$(\pi - \frac{1}{2}) r^2 \rho$	$\bar{x}$

(5)

(5)

$$\left(\pi - \frac{1}{2}\right) r^2 \rho g \, \bar{x} = \pi r^2 \rho g \, r - \frac{r^2}{2} \rho g \times \frac{3r}{2} \quad (10)$$

$$\left(\frac{2\pi - 1}{2}\right) \bar{x} = \left(\frac{4\pi - 3}{4}\right) r.$$

$$\bar{x} = \frac{4\pi - 3}{2(2\pi - 1)} r. \quad (5)$$

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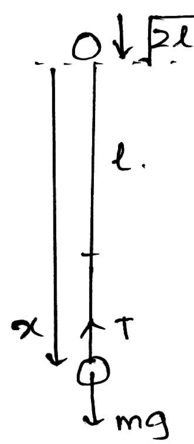
8)



P.E = 0

By the Law of Conservation of energy,

$$\frac{1}{2}\frac{2mg(x-l)^2}{l} - mgx = \frac{1}{2}m.2gl \quad (15)$$

$$\frac{x^2-2xl+l^2}{l} = l+x \quad (5)$$

$$x^2-2xl+l^2 = l^2+xl.$$

$$x = 3l. \ (x>0) \quad (5)$$

The maximum distance travelled by the particle is $3l$.

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9) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (5)

$$\frac{9}{10} = \frac{2}{3} + \frac{1}{2} - P(A \cap B) \quad (5)$$

$$P(A \cap B) = \frac{2}{3} + \frac{1}{2} - \frac{9}{10}$$

$$P(A \cap B) = \frac{4}{15} \quad (5)$$

$$P(A' \cap B) = P(B) - P(A \cap B) \quad (5)$$

$$= \frac{1}{2} - \frac{4}{15}$$

$$= \frac{7}{30} \quad (5)$$

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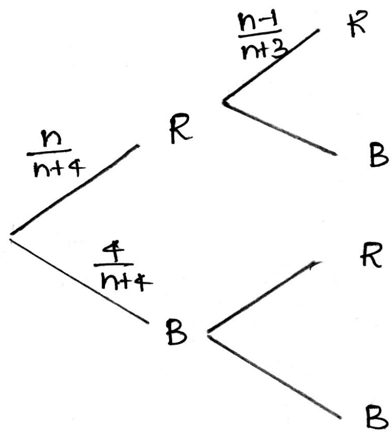
10)



$$\left(\frac{n}{n+4}\right)\left(\frac{n-1}{n+3}\right) = \frac{1}{3} \quad 10$$

$$3n^2 - 3n = n^2 + 7n + 12$$

$$n^2 - 5n + 6 = 0 \quad 5$$

$$(n-6)(n+1) = 0$$

$$n = 6 \quad \text{or} \quad n = -1 \quad 5$$

but $n \neq -1$, $\therefore$ $n = 6$ 5

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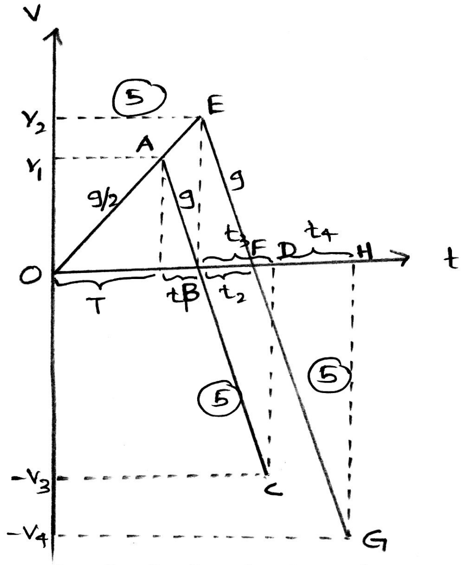
PART B.

11 a)



$\boxed{15}$

OA $\quad \frac{v_1}{T} = \frac{g}{2}$

$v_1 = \frac{gT}{2}$ 5

AB, $\quad \frac{v_1}{t_1} = g$

$t_1 = \frac{v_1}{g}$

$t_1 = \frac{gT/2}{g}$

$t_1 = \frac{T}{2}$. 5

Maximum height reached by the released part $=$ Area of OAB 5

$= \frac{1}{2}\left(T + \frac{T}{2}\right) v_1$

$= \frac{1}{2}\left(\frac{3T}{2}\right)\left(\frac{gT}{2}\right)$

$= \frac{3gT^2}{8}$ 5

$\boxed{20}$

OE $\quad \frac{v_2}{T + T/2} = \frac{g}{2}$

$\frac{v_2}{3T/2} = \frac{g}{2}$

$v_2 = \frac{3gT}{4}$ 5

ED. $\quad \frac{v_2}{t_2} = g$

$t_2 = \frac{v_2}{g}$

$t_2 = \frac{3T}{4}$ 5

The maximum beigh reached by the shuttle = Area of OED 5

$= \frac{1}{2}\left(T + \frac{T}{2} + \frac{3T}{4}\right)\frac{3gT}{4}$

$= \frac{1}{2}\left(\frac{9T}{4}\right)\left(\frac{3gT}{4}\right)$

$= \frac{27gT^2}{32}$ 5

$\boxed{20}$

Maximum height reached by released part = Distance travelled to reach the earth

Area of OAB = Area of BCD

$\frac{3gT^2}{8} = \frac{1}{2} t_3 V_3$ ——— (1) 5

BC $\quad \frac{V_3}{t_3} = g$

$t_3 = \frac{V_3}{g}$

By (1), $\quad \frac{3gT^2}{8} = \frac{1}{2}\frac{V_3}{g} \cdot V_3$

$V_3 = \frac{3g^2T^2}{4}$

$V_3 = \frac{\sqrt{3}gT}{2}$ 5 $\boxed{10}$.

Maximum height reached by shuttle = Distance travelled to reach earth.

Area of OEF = Area of FGH

$\frac{27gT^2}{32} = \frac{1}{2} t_4 V_4$ ——— (2), 5

FH $\quad \frac{V_4}{t_4} = g$

$t_4 = \frac{V_4}{g}$.

$\frac{27gT^2}{32} = \frac{1}{2} \cdot \frac{V_4}{g} \cdot V_4$

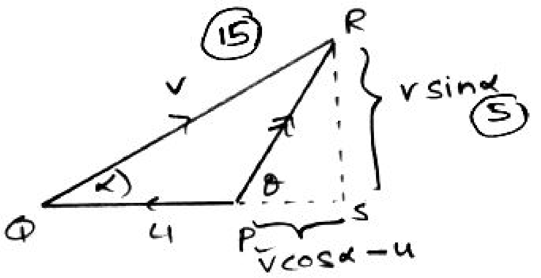
$V_4 = \frac{3\sqrt{3}gT}{4}$ 5 $\boxed{10}$

11) b) $V_{D,E} \rightarrow u$

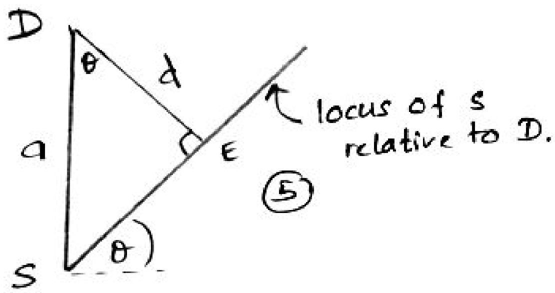
$V_{S,E}$ $\angle^{v}_{\alpha}$

$V_{S,D} = V_{S,E} + V_{E,D}$ (5)

$= \overrightarrow{QR}\ (v, \alpha) + \overleftarrow{QP}\ (u)$ (5)



$\boxed{30}$



$\boxed{05}$.

Shortest distance $DE = a\sin(90-\theta)$

$= a\cos\theta$. (5)

$= a \dfrac{v\cos\alpha - u}{\sqrt{(v\cos\alpha-u)^2+(v\sin\alpha)^2}}$ (10)

PRS Δ

$\cos\theta = \dfrac{v\cos\alpha - u}{\sqrt{(v\cos\alpha-u)^2+(v\sin\alpha)^2}}$ (5)

$= \dfrac{a(v\cos\alpha - u)}{\sqrt{v^2+u^2-2uv\cos\alpha}}$ (5)

$\boxed{25}$.

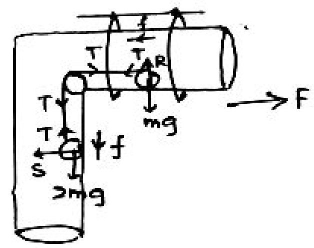
Time taken for shortest distance $\Big\}$ $= \dfrac{SE}{PR} = \dfrac{a\sin\theta}{PR}$ (5)

$= \dfrac{\dfrac{av\sin\alpha}{\sqrt{(v\cos\alpha-u)^2+(v\sin\alpha)^2}}}{\sqrt{(v\cos\alpha-u)^2+(v\sin\alpha)^2}}$ (10)

$= \dfrac{av\sin\alpha}{\sqrt{v^2+u^2-2uv\cos\alpha}}$ (5) $\boxed{20}$

12) b) For m, $\leftarrow F = ma$

$T = m(f - F)$ — ① (10)



For $2m$, $\downarrow F = ma$

$2mg - T = 2mf$ — ② (10)

For system, $\rightarrow F = ma$

$0 = MF + m(F - f) + 2mF$

$0 = (M + 3m)F - mf$ — ③ (10)

$a_{M,E} \rightarrow F$

$a_{m,M} \overset{\leftarrow}{f}$ $\quad a_{m,E}$

$a_{2m,M} \downarrow f$

$a_{m,E} = \overset{\leftarrow}{f} + \vec{F}$

$a_{2m,E} = \downarrow f + \vec{F}$ (10) $\boxed{40}$

① + ②

$2mg = 3mf - mF$

$2g = 3f - F$ — ④ (5)

$f = \dfrac{2g + F}{3}$ (5)

From ③

$0 = (M + 3m)F - m\left(\dfrac{2g + F}{3}\right)$ (5)

$0 = \left(M + 3m - \dfrac{m}{3}\right)F - \dfrac{2mg}{3}$

$\dfrac{2mg}{3} = \left(\dfrac{3M + 8m}{3}\right)F$

$F = \left(\dfrac{2mg}{3M + 8m}\right)$ (5)

From ③, $f = \dfrac{1}{m}(M + 3m)\left(\dfrac{2mg}{3M + 8m}\right)$

$= 2g\left(\dfrac{M + 3m}{3M + 8m}\right)$ (5) $\boxed{25}$

Acceleration of P. $\overset{\rightarrow F}{\downarrow f}$

$a = \sqrt{F^2 + f^2}$

$= \sqrt{\left(\dfrac{2mg}{3M + 8m}\right)^2 + \left(\dfrac{2(M + 3m)g}{3M + 8m}\right)^2}$ (5)

$= \dfrac{2g}{3M + 8m}\sqrt{M^2 + 10m^2 + 6Mm}$ (5) $\boxed{10}$

12) b)

Applying the Principle of Conservation of energy.

$$-Mgl - mga + \frac{1}{2}mkga =$$
$$- Mgl - mga\cos\theta + \frac{1}{2}mv^2 \quad (10)$$

$$-mga + \frac{1}{2}m\,kga = -mga\cos\theta + \frac{1}{2}mv^2$$

$$v^2 = ga(k-2+2\cos\theta) \quad (5)$$

Applying ~~lok~~ $F = ma$ for P,

$$R + T - mg\cos\theta = \frac{mv^2}{a} \quad (10)$$

$$R = -T + mg\cos\theta + \frac{m}{a}ga(k-2+2\cos\theta)$$

For the equlibrium of Q.

$$T - Mg = 0$$
$$T = Mg \quad (5)$$

$$\therefore \quad R = -Mg + mg\cos\theta + \frac{m}{a}\cdot ga(k-2+2\cos\theta) \quad (5)$$

$$= mg\left[k-2+3\cos\theta - \frac{M}{m}\right] \quad (5)$$

When $k = b$.

$$R = mg\left[b-2+3\cos\theta - \frac{M}{m}\right] \quad (5)$$
$$= mg\left[4 + 3\cos\theta - \frac{M}{m}\right]$$

$\boxed{45}$

When the reaction disappeared.

$$R = 0 \quad (5)$$

$$mg\left(4 + 3\cos\theta - \frac{M}{m}\right) = 0$$

$$\cos\theta = \frac{\frac{M}{m} - 4}{3} \quad (5)$$

But $\quad 0 < \theta < \pi.$ (5)

$$\cos 0 > \cos\theta > \cos\pi. \quad (5)$$

$$1 > \frac{\frac{M}{m} - 4}{3} > -1 \quad (5)$$

$$3 > \frac{M}{m} - 4 > -3$$

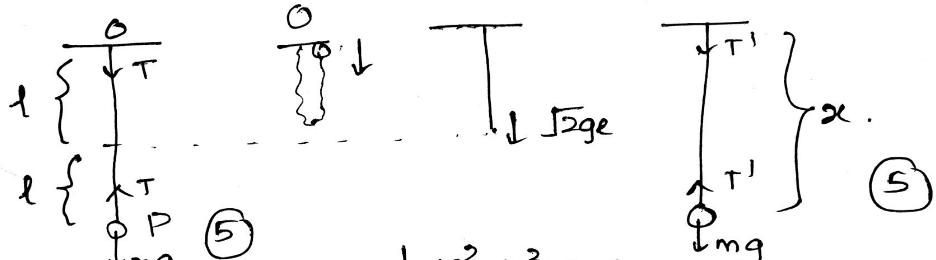
$$7 > \frac{M}{m} > 1$$

$$7m > M > m$$

The reaction is disappeared when

$7m > M > m.$ (5) $\quad \boxed{30}.$

13)



$\downarrow$ $v^2 = u^2 + 2as$

$v^2 = 0 + 2gl$ (10)

$v = \sqrt{2gl}$. (5)

$\boxed{15}$

$\uparrow$ $F = ma$

$T - mg = 0$ (10)

$\frac{\lambda l}{l} - mg = 0$ (5)

$\lambda = mg$ (5)

+ diagram. $\boxed{25}$

$\downarrow$ $F = ma$.

$mg - T' = m\ddot{x}$ (10)

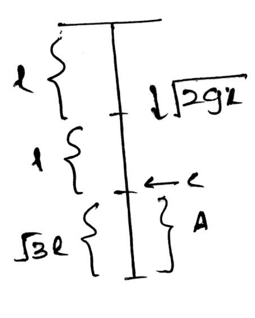
$mg - mg\left(\frac{x - l}{l}\right) = m\ddot{x}$ (5)

$-\frac{g}{l}(x - l - l) = \ddot{x}$

$-\frac{g}{l}(x - 2l) = \ddot{x}$ (5)

$\therefore$ The motion is simple harmonic. $\omega = \sqrt{\frac{g}{l}}$. (5)

+ diagram $\boxed{30}$



$\dot{x}^2 = \omega^2(A^2 - x^2)$.

$\dot{x} = \sqrt{2gl}$, $\omega = \sqrt{\frac{g}{l}}$, $x = l$. (10) conditions

$2gl = \frac{g}{l}(A^2 - l^2)$ (5)

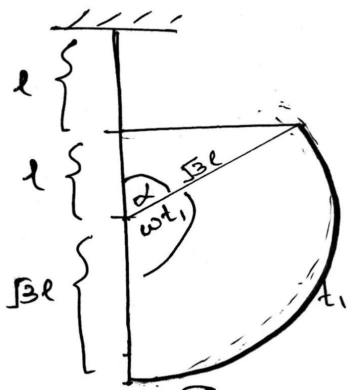
$2l^2 + l^2 = A^2$

$A^2 = 3l^2$

$A = \sqrt{3}\,l$. (5)

$\boxed{20}$

To calculate the time, let consider a horizontal circular motion of radius $\sqrt{3}l$ moving with ω angular velocity. (5)



(5)

$$\cos\alpha = \frac{l}{\sqrt{3}l} \quad (5)$$

$$\alpha = \cos^{-1}(1/\sqrt{3}) \quad (5)$$

$$\omega t_1 = \pi - \alpha. \quad (5)$$

$$= \pi - \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$t_1 = \frac{1}{\omega}\left(\pi - \cos^{-1}\frac{1}{\sqrt{3}}\right) \quad (5)$$

$$= \sqrt{\frac{l}{g}}\left\{\pi - \cos^{-1}\frac{1}{\sqrt{3}}\right\}$$

$\boxed{30}$

For the motion from O to distance l,

$\downarrow$ $v = u + at$

$$\sqrt{2gl} = 0 + gt_2 \quad (5)$$

$$t_2 = \sqrt{\frac{2l}{g}} \quad (5)$$

$\boxed{10}$

Time taken by particle to reach the maximum distance from O,

$$t_1 + t_2 = \sqrt{\frac{l}{g}}\left\{\pi - \cos^{-1}\frac{1}{\sqrt{3}}\right\} + \sqrt{\frac{2l}{g}}.$$

$$= \sqrt{\frac{l}{g}}\left\{\pi - \cos^{-1}\frac{1}{\sqrt{3}} + \sqrt{2}\right\} \quad (5)$$

Time taken to reach the maximum height is equat to the time taken by particle to return O. (10)

$\therefore$ Total time taken to reach O again $= 2\sqrt{\frac{l}{g}}\left\{\pi - \cos^{-1}\frac{1}{\sqrt{3}} + \sqrt{2}\right\}$ (5)

$\boxed{20}$

14) a) $\alpha \underline{a} + \beta \underline{b} = 0$

Let $\alpha \neq 0$. (5)

$\underline{a} + \frac{\beta}{\alpha} \underline{b} = 0$

$\underline{a} = -\frac{\beta}{\alpha} \underline{b}$ (5)

But $\underline{a}$ and $\underline{b}$ are non-zero, non parallel vectors.

$\therefore$ The above result is impossible.

then $\alpha = 0$ (5)

Let $\beta \neq 0$,

$\frac{\alpha}{\beta} \underline{a} + \underline{b} = 0$

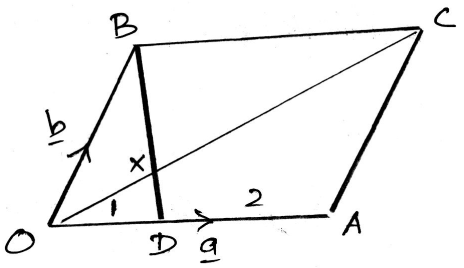
$\frac{\alpha \underline{a}}{\beta} = -\underline{b}$

As $\underline{a}$ and $\underline{b}$ are non-zero, non parallel vectors, above result is impossible (5) (similarly)

$\therefore \beta = 0$

$\therefore$ For $\alpha \underline{a} + \beta \underline{b} = 0$, $\alpha = 0$ and $\beta = 0$ (5)

$\boxed{25}$



$\overrightarrow{OA} = \underline{a}$

$\overrightarrow{OB} = \underline{b}$,

$\left.\begin{array}{l} \left.\begin{array}{l} OX = \lambda OC \\ OX \parallel OC \end{array}\right\} \overrightarrow{OX} = \lambda \overrightarrow{OC} \\ \left.\begin{array}{l} BX = \mu BD \\ BX \parallel BD \end{array}\right\} \overrightarrow{BX} = \mu \overrightarrow{BD} \end{array}\right\}$ (5)

$\overrightarrow{OX} = \lambda \overrightarrow{OC}$

$\overrightarrow{OX} = \lambda(\overrightarrow{OA} + \overrightarrow{AC})$ 5

$\overrightarrow{OX} = \lambda(\underline{a} + \underline{b})$ 5 ——— (1)

$\overrightarrow{OX} = \overrightarrow{OB} + \overrightarrow{BX}$ 5

$= \overrightarrow{OB} + \mu \overrightarrow{BD}$

$= \overrightarrow{OB} + \mu(\overrightarrow{BO} + \overrightarrow{OD})$

$= \underline{b} + \mu(-\underline{b} + \frac{1}{3}\underline{a})$

$= (1-\mu)\underline{b} + \frac{\mu}{3}\underline{a}$ ——— (2) 5

By (1) and (2).

$\lambda(\underline{a} + \underline{b}) = (1-\mu)\underline{b} + \frac{\mu}{3}\underline{a}$

$(\lambda - \frac{\mu}{3})\underline{a} + (\lambda - 1 + \mu)\underline{b} = 0$ 5 $\boxed{30}$

As $\underline{a}$ and $\underline{b}$ are non-parallel and non-zero vectors.

$\lambda - \frac{\mu}{3} = 0$ and $\lambda - 1 + \mu = 0$

$\lambda = \frac{\mu}{3}$ — (1) and $\lambda + \mu = 1$ — (2) 5

By (1) and (2).

$\frac{\mu}{3} + \mu = 1$

$4\mu = 3$

$\mu = 3/4$ 5

$\lambda = 1/4$ 5

$\therefore OX = \frac{1}{4} OC$

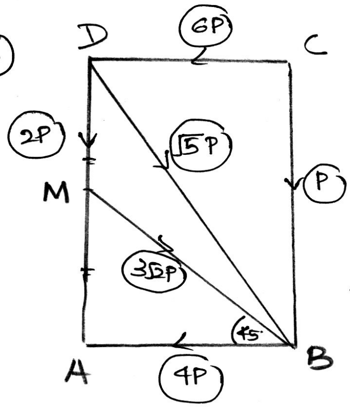
$OX : XC = 1 : 3$ 5

$BX = \frac{3}{4} BD$.

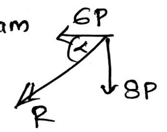
$BX : XD = 3 : 1$ 5

$\boxed{25}$

b)



(5) diagram



$\overleftarrow{X} = 4P + 6P - 3P - P = 6P$ (15) (5)

$\downarrow Y = P + 2P + 3P + 2P$ (10)

$= 8P$ (5)

$R = \sqrt{(6P)^2 + (8P)^2}$

$= 10P$ (5)

$\tan\alpha = \dfrac{8P}{6P} = \dfrac{4}{3}$ (5)

$\overset{\frown}{A},\ G. = P.a - 6P.2a + 3\sqrt{2}P.\dfrac{a}{\sqrt{2}} + \sqrt{5}P.a.\dfrac{2}{\sqrt{5}}$ (10)

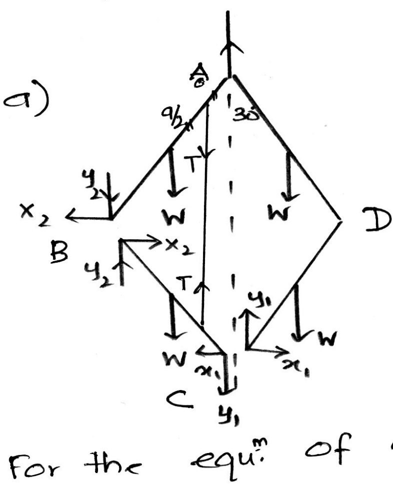
$= P.a - 12Pa + 3Pa + 2Pa.$ (5)

$= -6Pa$ (5)

There is a couple of moment 6Pa in anticlockwise (5)

$\boxed{70}$

(15) a)



For the equ^m of AD and CD,

$\overset{\frown}{A},$ (10)

$x_1 \times 4a\cos 30 - 2W.a\sin 30 = 0$

$x_1 . 4a . \dfrac{\sqrt{3}}{2} = 2W.a.\dfrac{1}{2}$

$x_1 = \dfrac{W}{2\sqrt{3}}$ (5)

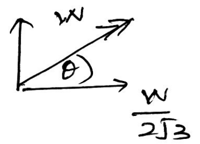
For the equ^m of CD.

$\overset{\frown}{D},\ y_1 \times 2a\sin 30 - x_1 \times 2a\cos 30 - W \times a\sin 30 = 0$ (10)

$y_1 \times 2a.\dfrac{1}{2} - x_1 \times 2a.\times\dfrac{\sqrt{3}}{2} - W.a.\dfrac{1}{2} = 0$

$y_1 = W.$ (5)

Reaction at C. $= \sqrt{\left(\frac{W}{2\sqrt{3}}\right)^2 + W^2}$



$= \frac{W}{2}\sqrt{\frac{13}{3}}$ (5)

$\tan\theta = \frac{W}{W/2\sqrt{3}}$

$\theta = \tan^{-1}(2\sqrt{3})$ (5)

ii) For the equ^m of BC.

$\overset{\curvearrowleft}{B}$ $T\times\frac{3a}{2}\sin 30^\circ - W.\times a\sin 30^\circ - x_1\times 2a\cos 30 - y_1\times 2a\sin 30^\circ = 0$ (10)

$T\times\frac{3a}{2}\times\frac{1}{2} = W.\frac{a}{2} + \frac{W}{2\sqrt{3}}\times 2a\times\frac{\sqrt{3}}{2} + W\times 2a\times\frac{1}{2}$

$\frac{3T}{4} = \frac{W}{2} + \frac{W}{2} + W$

$\frac{3T}{4} = 2W$

$T = \frac{8W}{3}$ (5)

iii). For the equ^m of BC,

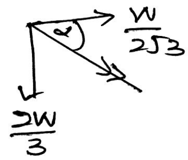
$\rightarrow$ $x_2 = x_1 = \frac{W}{2\sqrt{3}}.$

$\uparrow$ $y_2 = -T + y_1 + W.$

$y_2 = -\frac{8W}{3} + 2W$

$y_2 = -\frac{2W}{3}$

Reaction at B $= \sqrt{\left(\frac{W}{2\sqrt{3}}\right)^2 + \left(\frac{2W}{3}\right)^2}$



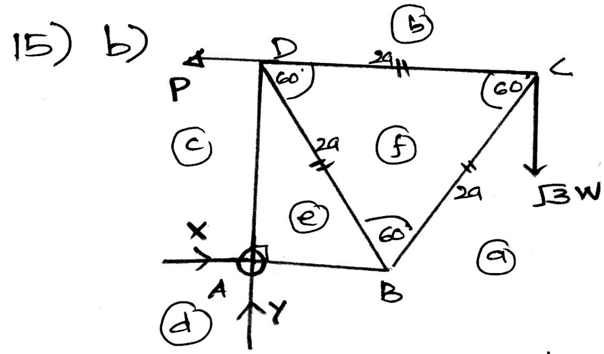
$= \frac{\sqrt{19}}{6}W$ (5)

$\tan\alpha = \frac{2W/3}{W/2\sqrt{3}}$

$\boxed{65}$

$\alpha = \tan^{-1}\left(\frac{4}{\sqrt{3}}\right)$ (5)

15) b)



For the eq^m of system,

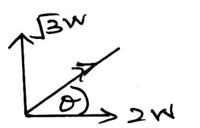
$A \curvearrowleft \quad P \times 2a \cos 30^\circ = \sqrt{3} W \times 2a.$

$P \cdot \frac{\sqrt{3}}{2} = \sqrt{3} W$

$P = 2W$ (5)

$\rightarrow \quad X = P = 2W.$

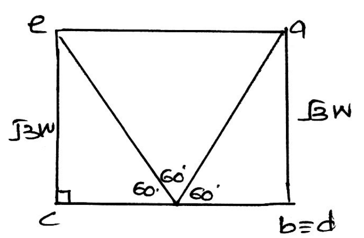
$\uparrow \quad Y = \sqrt{3} W.$



Reaction at A $= \sqrt{(2W)^2 + (\sqrt{3}W)^2}$ (5)

$\tan\theta = \frac{\sqrt{3}}{2}.$

$\theta = \tan^{-1} \frac{\sqrt{3}}{2}$ (5)



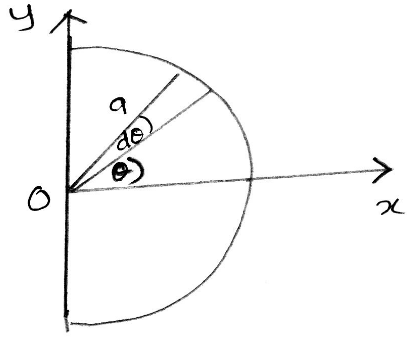
(20)

rod	Magnitude	Tension/Thrust
AB (ea)	2W	Thrust
AD (ce)	$\sqrt{3}W$	Thrust
BD (ef)	2W	Tension
BC (af)	2W	Thrust
CD (bf)	W	Thrust

(25) (25)

85

(16)



By symmetry, the center of mass lies on Ox axis. (5)

ρ is the mass per unit area.

$$x = \frac{2a}{3}\cos\theta$$

$$dm = \frac{1}{2}a^2 d\theta \rho.$$

$$\bar{x} = \frac{\int_{-\pi/2}^{\pi/2} \frac{2a}{3}\cos\theta \times \frac{1}{2}a^2 d\theta\rho.}{\int_{-\pi/2}^{\pi/2} \frac{1}{2}a^2 d\theta\rho.} \quad (5) \quad (5)$$

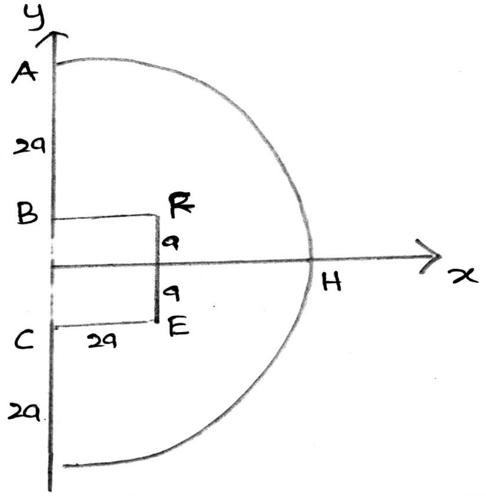
$$= \frac{2a}{3} \frac{\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta}{\int_{-\pi/2}^{\pi/2} d\theta.}$$

$$= \frac{2a}{3} \frac{\left[\sin\theta\right]_{-\pi/2}^{\pi/2}}{\left[\theta\right]_{-\pi/2}^{\pi/2}} \quad (5) \quad (5)$$

$$= \frac{2a}{3} \times \frac{2}{\pi}$$

$$= \frac{4a}{3\pi} \quad (5)$$

$\boxed{30}$



By symmetry, the center of mass lies on the Ox axis. 5

ρ is the mass per unit area.

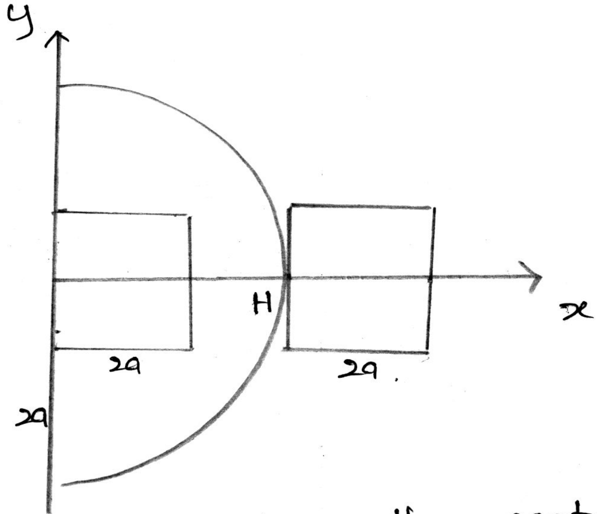
Object	Mass	Distance from O
D	$\frac{\pi(3a)^2}{2} = \frac{9\pi a^2\rho}{2}$ 5	$\frac{4(3a)}{3\pi} = \frac{4a}{\pi}$. 5
□	$4a^2\rho$ 5	a 5
Ↄ	$\left(\frac{9\pi-8}{2}\right)a^2\rho$ 5	$\bar{x}$

$$\left(\frac{9\pi-8}{2}\right)a^2\rho\ \bar{x} = \frac{9\pi a^2}{2}\rho \times \frac{4a}{\pi} - 4a^2\rho \times a \quad 10$$

$$\left(\frac{9\pi-8}{2}\right)\bar{x} = (18-4)a$$

$$\bar{x} = \frac{28a}{9\pi-8} \quad 5$$

45



By symmetry, the center of mass lies on Ox axis. 5

Object	mass	Distance from O	
	$\left(\frac{9\pi-8}{2}\right)a^2\rho$	$\frac{28a}{9\pi-8}$	5
	$4a^2\rho$.	$4a$	5
	$\frac{9\pi a^2}{2}\rho$. 5	$\bar{\bar{x}}$	

$$\frac{9\pi a^2}{2}\rho\,\bar{\bar{x}} = \left(\frac{9\pi-8}{2}\right)a^2\rho\left(\frac{28a}{9\pi-8}\right) + 4a^2\rho \times 4a$$

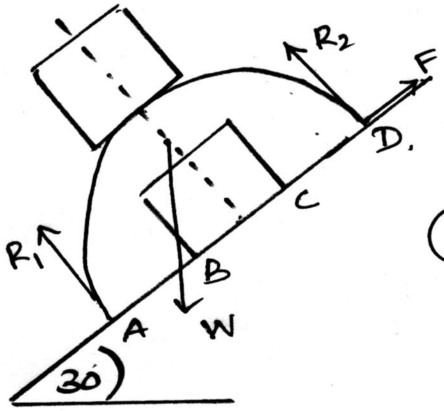
10

$$\frac{9\pi}{2}\bar{\bar{x}} = 14a + 16a. \quad 5$$

$$\bar{\bar{x}} = \frac{60}{9\pi}a$$

$$\bar{\bar{x}} = \frac{20}{3\pi}a \quad 5$$

$\boxed{40}$



(10)

$\nearrow$ $F = W \sin 30°$ (5)

$\nwarrow$ $R_1 + R_2 = W \cos 30°$ (5)

For the equilibrium,

$$\mu \geqslant \left| \frac{F}{R_1 + R_2} \right| \quad (10)$$

$$\mu \geqslant \frac{W \sin 30}{W \cos 30}$$

$$\mu \geqslant \tan 30°$$

$$\mu \geqslant \frac{1}{\sqrt{3}} \quad (5)$$

$\boxed{35}$

17) a) i) If $A \cap B = \phi$, then A and B are mutually exclusive events. 5

ii) If $A \cup B = \Omega$, then A and B are exhaustive. 5

$\boxed{10}$

b). $P(A) + P(B) + P(C) = P(\Omega)$ 5

$2a^2 + 2a + 8a - 1 = 1$ 5

$2a^2 + 10a - 2 = 0$

$a^2 + 5a - 1 = 0$ 5

$\left(a + \frac{5}{2}\right)^2 = 1 + \frac{25}{4}$

$a + \frac{5}{2} = \pm \frac{\sqrt{29}}{2}$

$a = -\frac{5}{2} \pm \frac{\sqrt{29}}{2}$ 5

As $a > 0$, $a = \frac{\sqrt{29} - 5}{2}$ 5

$\boxed{25}$

c) i) $A = (A \cap B) \cup (A \cap B')$ 5

$P(A) = P[(A \cap B) \cup (A \cap B')]$

$P(A) = P(A \cap B) + P(A \cap B')$ 5 — 1

$(\because (A \cap B) \cap (A \cap B') = \phi)$ 5

$\boxed{15}$

ii) $A \cup B = (A \cup B') \cup B$ (5)

As $(A \cap B') \cap B = \phi$

$P(A \cup B) = P(A \cup B') + P(B)$ —— (1)

(5)

By (1) and (2),

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (5)

$\boxed{15}$

$P(A) = P(A \cap B) + P(A \cap B')$

$\frac{3}{4} = P(A \cap B) + \frac{2}{5}$ (5)

$P(A \cap B) = \frac{3}{4} - \frac{2}{5} = \frac{7}{20}$ (5)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{4} + \frac{1}{2} - \frac{7}{20}$ (5)

$= \frac{15 + 10 - 7}{20}$

$= \frac{9}{10}$ (5)

$P(A' \cap B) = P(B) - P(A \cap B)$

$= \frac{1}{2} - \frac{7}{20}$ (5)

$= \frac{3}{20}$ (5)

$P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$ (5)

$= \frac{1}{4} + \frac{1}{2} - \frac{3}{20}$ (5)

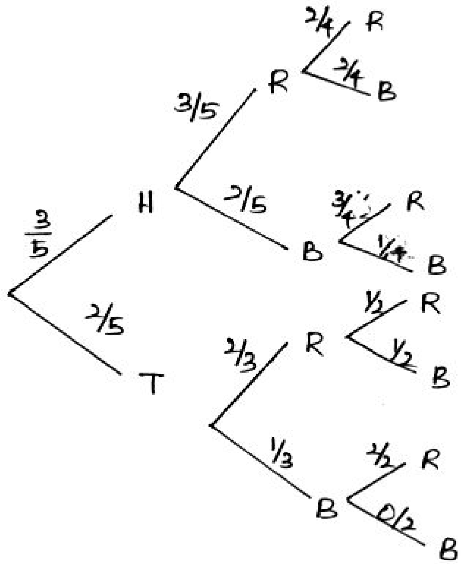
$= \frac{3}{5}$ (5)

$\therefore \quad P(A' \cup B') = 1 - P(A \cap B)$ (5)

$= 1 - \frac{7}{20}$

$= \frac{13}{20}$ (5) $\qquad \boxed{55}$

d)



i) $\frac{3}{5} \times \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{2}{3} \times \frac{1}{2}$ (10)

$= \frac{18}{100} + \frac{2}{15}$

$= \frac{47}{150}$ (5)

ii) $\frac{2}{5} \times \frac{2}{3} \times \frac{1}{2} + \frac{2}{5} \times \frac{1}{3} \times \frac{2}{2}$ (10)

$= \frac{4}{15}$ (5)

$\boxed{30}$