1). $(1-k)x^2 + x + k = 0$

Consider,

$\Delta x = b^2 - 4ac,$

$= 1 - 4(1-k)k$ 5

$= 4k^2 - 4k + 1$

$= 4\left\{\left(k-\frac{1}{2}\right)^2 + \frac{15}{4}\right\} > 0$ 5

Roots of the eq^n α and β,

$\alpha + \beta = \frac{-1}{1-k}$

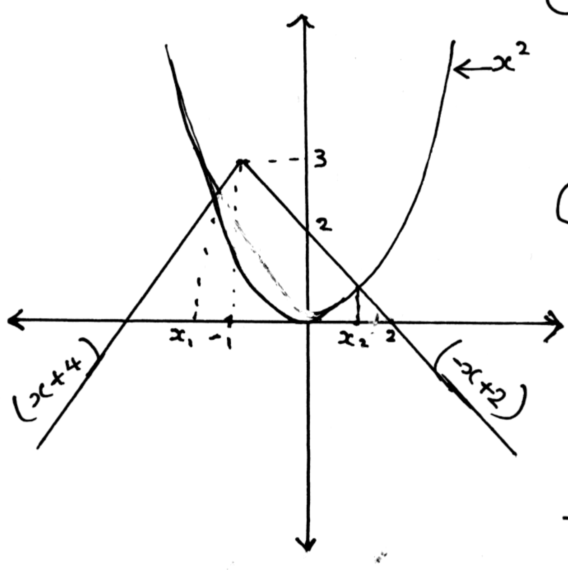
5 $= \frac{1}{k-1} < 0 \quad (\because 0 < k < 1)$

25

$\alpha\beta = \frac{k}{1-k} > 0$ 5

Therfore, eq^n has negative real roots. 5

2) $3 - |x+1| < x^2$



5 $\underline{x_1}$

$x^2 = x+4$

$x^2 - x - 4 = 0$

$x = \frac{1 \pm \sqrt{17}}{2}$

5 $\therefore x_1 = \frac{1-\sqrt{17}}{2}$

$\underline{x_2}$ 5

$x^2 = 2-x$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x_2 = 1 \; ; x_2 \neq -2$

5

solution.

$x < \frac{1-\sqrt{17}}{2}$ or $x > 1$

25

5

03) $3^{2x+1} - 3^{x+4} + 3^3 = 3^x$

$3(3^x)^2 - 3^4(3^x) - 3^x + 3^3 = 0$

$3 \cdot (3^x)^2 - 82(3^x) + 27 = 0$ (5)

Let $3^x = t$,

(5) $3t^2 - 82t + 27 = 0$

$(t^2 - 2)(3t-1) = 0$ (5)

$t = 27$ or $t = 1/3$

$3^x = 27$, $3^x = 3^{-1}$

$\underline{\underline{x = 3}}$ (5) $\underline{\underline{x = -1}}$ (5)

25

04) $\lim_{x \to 0} \dfrac{\sin(\pi \cos^2 x)}{x^2}$

$\lim_{x \to 0} \dfrac{\sin[\pi(1 - \sin^2 x)]}{x^2}$ (5)

$\lim_{x \to 0} \dfrac{\sin \pi \sin^2 x}{x^2}$ (5)

$\lim_{x \to 0} \dfrac{\sin \pi \sin^2 x}{\pi \sin^2 x} \times \dfrac{\pi \sin^2 x}{x^2}$ (5)

$= \lim_{x \to 0} \dfrac{\sin \pi \sin^2 x}{\pi \sin^2 x} \times \pi \lim_{x \to 0} \dfrac{\sin^2 x}{x^2}$

$= 1 \times \pi \times (1)^2$ (5)

$= \underline{\underline{\pi}}$ (5)

25

05). $\log_3 x + \log_3 y = 3$ —— (1)

$\log_y x = 2$ —— (2)

From (1),

$\log_3 xy = 3$ (5)

$xy = 27$ —— (3)

From (2);

$x = y^2$ —— (4)

(5)

(3) and (4), (5)

$y^3 = 27$

$\underline{\underline{y = 3}}$ (5) $\underline{\underline{x = 9}}$ (5)

25

06). Let; $f(x) = x^3 + ax^2 + b$

$g(x) = ax^3 + bx^2 + x - a$

Let, $(x-\alpha)$, common factor of $f(x)$ and $g(x)$

$\alpha^3 + a\alpha^2 + b = 0$ —— (1) (5)

$a\alpha^3 + b\alpha^2 + \alpha - a = 0$ —— (2) (5)

$a \times (1) - (2) \Rightarrow$

(5) $(a^2 - b)\alpha^2 - \alpha + ab + a = 0$

$(b - a^2)\alpha^2 + \alpha - a(1+b) = 0$ (5)

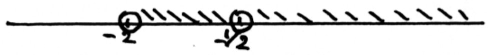
$\Rightarrow (b-a^2)x^2 + x - a(1+b)$ has common linear factor $(x-\alpha)$. (5)

25

2

07). $f(x) = \dfrac{1}{\sqrt{x+2}}$; $x \geq -2$ $\quad g(x) = 2x+1$,

(i) $\dfrac{f}{g} = \dfrac{1}{\sqrt{x+2}\,(2x+1)}$ 5



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Domain of $\dfrac{f}{g}$; $\underline{\underline{(-2, -\tfrac{1}{2}) \cup (-\tfrac{1}{2}, \infty)}}$ 5

ii). $\left(\dfrac{f}{g}\right)(0) = \dfrac{1}{\sqrt{0+2}\,(2\times 0+1)}$ 5 $= \underline{\underline{\dfrac{1}{\sqrt{2}}}}$ 5

25

08). $x^2 - 6ax + 2 - 2a + 9a^2 = 0$ $\langle$ α, β

roots of the eq^n,

$$x = \frac{6a \pm \sqrt{36a^2 - 4(2-2a+9a^2)}}{2}$$ 5

$$x = \frac{6a \pm \sqrt{8a-8}}{2}$$

$$= 3a \pm \sqrt{2a-2}$$ 5

But, $\alpha, \beta > 3$ $\qquad \alpha+\beta > 6$,

$a > 1$

$\therefore\ 3a - \sqrt{2a-2} > 3$ 5

$3(a-1) > \sqrt{2a-2}$

$9(a-1)^2 > 2a-2$

$9a^2 - 20a + 11 > 0$

5 $(9a-11)(a+1) > 0$

$(9a-11) > 0$

$\underline{\underline{a > 11/9}}$ 5

25

9) $\sec\alpha + \tan\alpha = p$

5 $\tan\alpha = p - \sec\alpha$

$\Longrightarrow \tan^2\alpha = (p - \sec\alpha)^2$ 5

$(\tan\alpha - p)^2 = 1 + \tan^2\alpha$ 5

5

$\cancel{\tan^2\alpha} - 2p\tan\alpha + p^2 = 1 + \cancel{\tan^2\alpha}$

$$\underline{\underline{\tan\alpha = \frac{p^2 - 1}{2p}}}$$ 5

25

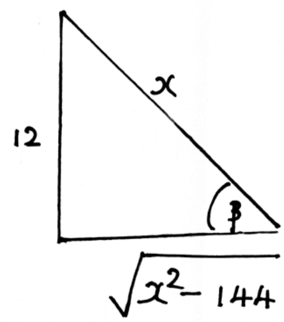
10) $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$

Let,

$\alpha = \sin^{-1}\left(\frac{5}{x}\right)$, $\beta = \sin^{-1}\left(\frac{12}{x}\right)$, γ

5

$\sin\alpha = \frac{5}{x}$ $\sin\beta = \frac{12}{x}$



Then, $\alpha + \beta = \frac{\pi}{2}$

$\alpha = \frac{\pi}{2} - \beta$ 5

$\sin\alpha = \sin\left(\frac{\pi}{2} - \beta\right)$

$\sin\alpha = \cos\beta$ 5

$\frac{5}{x} = \frac{\sqrt{x^2 - 144}}{x}$; $x \neq 0$,

25

$x^2 - 144 = 25$ $x = \pm 13$ $(\alpha, \beta < \pi/2)$

5

solution $\underline{\underline{x = 13}}$ 5

3

PART - B

(11). a). $f(x) = x^4 + px^2 + r$; $f(1) = -9$, $f(0) = -8$

$f(1) = 1 + p + r = -9$

$f(0) = \underline{\underline{r = -8}}$ 5

$\underline{\underline{p = -2}}$ 5

$$x^4 - 2x^2 - 8 \equiv (ax^2+b)^2 + c$$
$$= a^2x^4 + 2abx^2 + b^2 + c$$

Equating coefficients

$(a > 0)$

$x^4 \longrightarrow 1 = a^2$ — (1)

$x^2 \longrightarrow 0 = 2ab$

$ab = -1$ — (2) 10

$-8 = b^2 + c$ — (3)

5 $\underline{a = 1}$ $\underline{\underline{b = -1}}$ 5 $\underline{\underline{c = -9}}$ 5

$$\therefore f(x) = (x^2-1)^2 - 9$$
$$= (x^2-1)^2 - 3^2 \quad 5$$
$$= (x^2-1-3)(x^2-1+3)$$
$$= (x^2-4)(x^2+2) \quad 5$$
$$= (x-2)(x+2)(x^2+2)$$

55

Hence; real roots of the eq^n 5 $\underline{\underline{x = 2}}$ $\underline{\underline{x = -2}}$ 5

11

b) Let $h(x) = (p-1)x^2 - 4x + p - 1$

$\forall\, x \in \mathbb{R}, \quad h(x) > 0;$

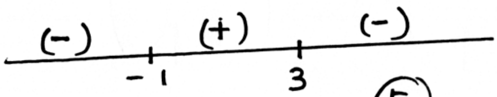
$(p-1) > 0$ 5 and $\Delta_x < 0$ 5

$\Delta_x = 16 - 4(p-1)(p-1) < 0$

$4 - (p-1)^2 < 0$ 5

$(2-p+1)(2+p-1) < 0$

5 $(3-p)(p+1) < 0$



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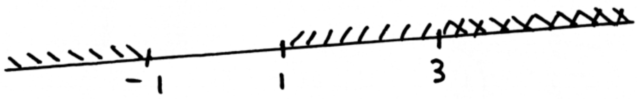
$\boxed{30}$

Then,



$\therefore \quad p > 3$ 5

c) $ax^2+bx+c=0 \;\Big\langle\; \begin{matrix}\alpha \\ \beta\end{matrix}$ 5 $\quad \alpha+\beta = -b/a$, $\quad \alpha\beta = c/a$

$cx^2 - 2bx + 4a = 0 \;\Big\langle\; \begin{matrix}\lambda \\ \mu\end{matrix}$.

$$x = \frac{2b \pm \sqrt{4b^2 - 4.c.4a}}{2c} = \frac{b \pm \sqrt{b^2 - 4ac}}{c}$$ 5

$$x = \frac{b}{c} \pm \sqrt{\frac{b^2}{c^2} - 4\frac{a}{c}}$$

$$= -\left(\frac{\alpha+\beta}{\alpha\beta}\right) \pm \sqrt{\left(\frac{\alpha+\beta}{\alpha\beta}\right)^2 - \frac{4}{\alpha\beta}} \quad (10)$$

$$= -\left(\frac{\alpha+\beta}{\alpha\beta}\right) \pm \sqrt{\frac{(\alpha+\beta)^2\alpha - 4\alpha\beta}{\alpha\beta}} \quad (5)$$

$$= -\left(\frac{\alpha+\beta}{\alpha\beta}\right) \pm \left(\frac{\alpha-\beta}{\alpha\beta}\right)$$

(5)

$$\therefore \quad \lambda = -\left(\frac{\alpha+\beta}{\alpha\beta}\right) + \left(\frac{\alpha-\beta}{\alpha\beta}\right)$$

$$\lambda = \frac{-2}{\alpha} \quad (5)$$

$$\mu = \frac{-2}{\beta} \quad (5)$$

40

d). $$\frac{x^2-1}{x^2(2x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(2x+1)} \quad (10)$$

$$x^2 - 1 = Ax(2x+1) + B(2x+1) + Cx^2$$

$x = -\frac{1}{2}$ $\qquad \frac{1}{4} - 1 = \frac{C}{4}$ $\qquad$ $x = 0$

$-1 = B$ (5)

(5) $C = -3$

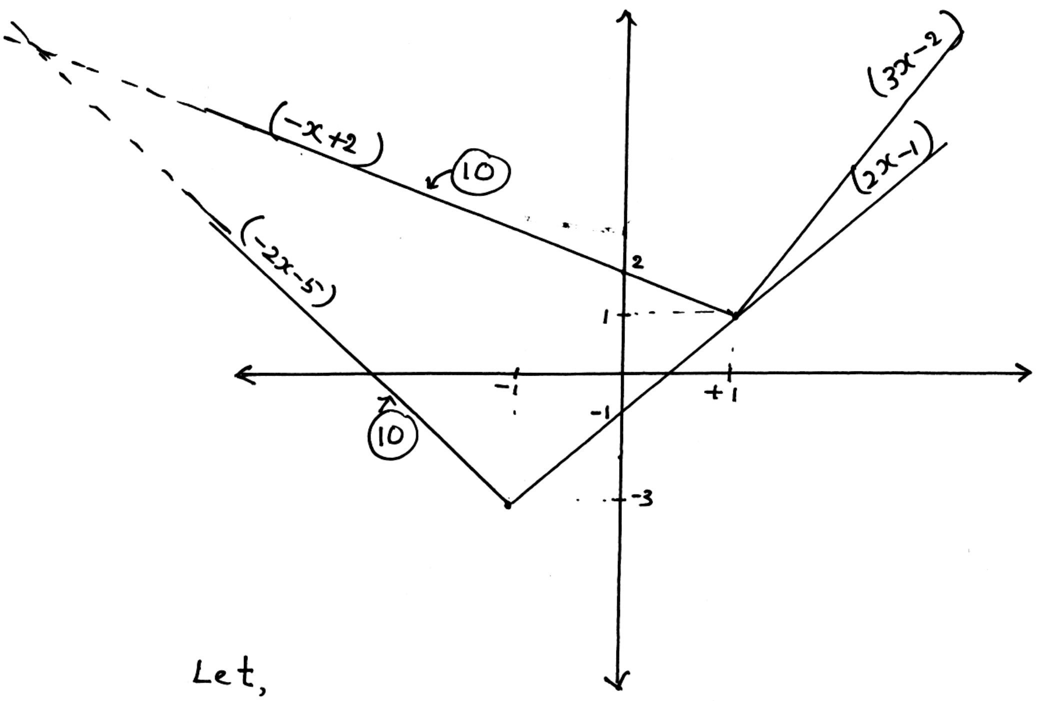
$x = 1$ $\qquad 0 = A(3) - 1(3) - 3 \qquad A = 2$

25

$$\therefore \frac{x^2-1}{x^2(2x+1)} = \frac{2}{x} - \frac{1}{x^2} - \frac{3}{(2x+1)} \quad (5)$$

12. a). $y = 2|x+1| - 3$ $\qquad\qquad$ $y = x + 2|x-1|$

$$y = \begin{cases} 2(x+1)-3 \,;\ x \geq -1 \\ 2x-1 \\ -2(x+1)-3 \,;\ x < -1 \\ -2x-5 \end{cases} \quad 5 \qquad\qquad y = \begin{cases} x+2(x-1) \,;\ x \geq 1 \\ 3x-2 \\ x-2(x-1) \,;\ x < 1 \\ -x+2 \end{cases} \quad 5$$



Let,

$x + 2|x-1| = 2|x+1| - 3$;

$\underline{\underline{x = 1}}$ 5

$-x+2 = -2x-5$

$\underline{x = -7}$ 5

Let,

$x + 2|x-1| > 2|x+1| - 3$;

5 $\underline{x > -7}$; $x \neq 1$ 5

50

5

b). $a = \log_{2n} n \qquad b = \log_{3n} 2n \qquad c = \log_{4n} 3n$

Considering;

$1 + abc = 1 + \log_{2n} n \cdot \log_{3n} 2n \; \log_{4n} 3n$ 5

$= 1 + \dfrac{\log n}{\cancel{\log 2n}} \times \dfrac{\cancel{\log 2n}}{\cancel{\log 3n}} \times \dfrac{\cancel{\log 3n}}{\log 4n}$ 10

$= 1 + \dfrac{\log n}{\log 4n}$ 5

$= 1 + \log_{4n} n$

$= \log_{4n} 4n + \log_{4n} n$ 5

$= \log_{4n} 4n^2$ 5 $= \log_{4n} (2n)^2$ 5

$= 2 \log_{4n} 2n$ 5

$= 2 \log_{3n} 2n \times \log_{4n} 3n$ 5

$\underline{\underline{1 + abc = 2bc}}$ 5

50

12)

e) $a^x = b^y = c^z = d^w = t$ 5 + 5

$x\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right) = \log_a bcd$

Considering;

$x\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right)$

$= \log_a t \left(\frac{1}{\log_b t} + \frac{1}{\log_c t} + \frac{1}{\log_d t}\right)$ 10

5

$= \log_a t \log_t (bcd)$ 10

$= \cancel{\log_a} t \dfrac{\log_a bcd}{\cancel{\log_a} t}$ 10

$= \log_a bcd$

5

$\therefore \quad x\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right) = \log_a bcd$

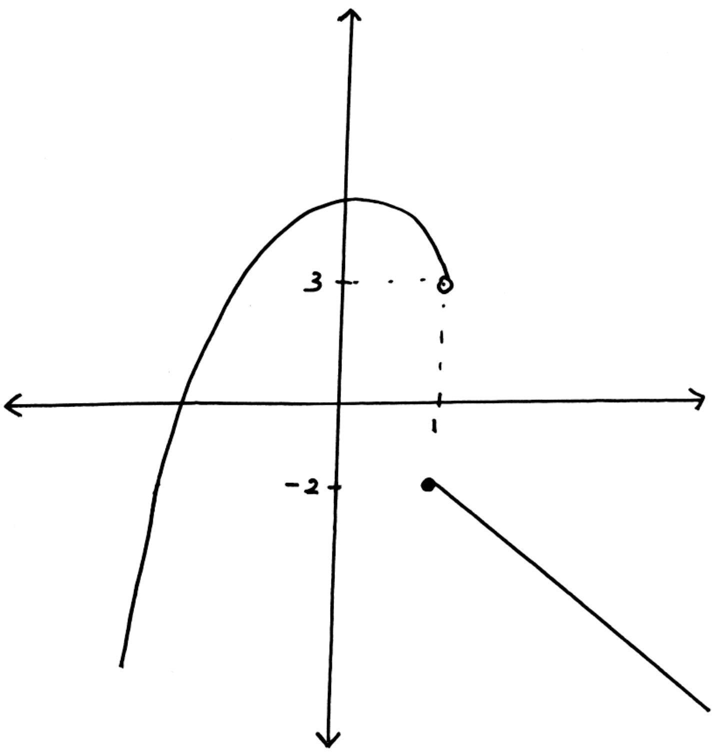
50

6

13) $f : \mathbb{R} \longrightarrow \mathbb{R}$

$$f(x) = \begin{cases} -x^2+4 \ ; & x<1 \\ -2x \ ; & x \geq 1 \end{cases}$$

i)



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ii). $\lim\limits_{x \to 1^-} f(x) = 3$ 5 $\qquad \lim\limits_{x \to 1^+} f(x) = -2$ 5

iii). $\lim\limits_{x \to 1^-} f(x) \neq \lim\limits_{x \to 1^+} f(x)$ 10

5

Therfore, not continuous at the point $x=1$.

iv). $\lim\limits_{x \to 1} f(x)$, does not exist. 10

55

b). $\lim_{a \to 0} \frac{\sin a}{a} = 1$ To proof 40

I). $\lim_{x \to 3} \frac{\sqrt{2x-1} - \sqrt{5}}{\sin(x-3)}$

$= \lim_{x \to 3} \frac{(2x-1-5)}{\sin(x-3)\left(\sqrt{2x-1}+\sqrt{5}\right)}$ 5

$= \lim_{x \to 3} \frac{2(x-3)}{\sin(x-3)\left(\sqrt{2x-1}+\sqrt{5}\right)}$

$= 2\lim_{x \to 3} \frac{(x-3)}{\sin(x-3)}$ 5 $\lim_{x \to 3} \frac{1}{\left(\sqrt{2x-1}+\sqrt{5}\right)}$ 5

$= 2 \times 1 \times \frac{1}{2\sqrt{5}}$ 5

$= \frac{1}{\sqrt{5}}$ 5 25

ii). $\lim_{x \to 0} \frac{\left(\sqrt{4+x^2}-2\right)\left(1-\cos 2x\right)}{x^4}$

$= \lim_{x \to 0}$ 5 $\frac{(4+x^2-2)}{(\sqrt{4+x^2}+2)} \quad \frac{2\sin^2 x}{x^4}$ 5

$= 2\lim_{x \to 0} \left(\frac{\sin^2 x}{x^2}\right)$ 5 $\lim_{x \to 0} \frac{1}{(\sqrt{4+x^2}+2)}$ 5

$= 2 \times (1)^2 \times \frac{1}{4}$ 5

$= \frac{1}{2}$ 5

30

14). (a). $ax^2+bx+c=0 \left\langle \begin{matrix}\alpha \\ \beta\end{matrix}\right.$

For positive real roots,

$\Delta x = b^2 - 4ac \geq 0$

$\alpha+\beta > 0$

$\alpha\beta > 0$ 15

$a^2x^2 + a(3b-2c)x + (2b-c)(b-c) + ac = 0$

$\Delta x = b^2 - 4ac \geq 0$, — (1) 5

$\alpha+\beta = -b/a > 0$ — (2) 5 $\quad \alpha\beta = c/a > 0$ — (3) 5

$$a^2x^2 + a(3b-2c)x + (2b-c)(b-c) + ac = 0 \begin{smallmatrix}\lambda \\ \mu\end{smallmatrix} \quad \text{— (A)}$$

$$\Delta_x = a^2(3b-2c)^2 - 4a^2\{(2b-c)(b-c) + ac\} \quad (10)$$

$$= a^2\{9b^2 - 12bc + 4c^2 - 4(2b^2 - 3bc + c^2 + ac)\}$$

$$= a^2\{b^2 - 4ac\} \geq 0 \quad (5) \quad (\text{From } (1))$$

$$\lambda + \mu = \frac{a(2c-3b)}{a^2}$$

$$= \frac{(2c-3b)}{a} \quad (10)$$

$$= 2\left(\frac{c}{a}\right) - 3\left(\frac{b}{a}\right) > 0 \quad ((2) \text{ and } (3))$$

$$\lambda\mu = \frac{(2b-c)(b-c)+ac}{a^2} \quad (5)$$

$$= \frac{2b^2 - 3bc + c^2 + ac}{a^2} \quad (5)$$

$$= 2\left(\frac{b}{a}\right)^2 - 3\underset{(-)}{\left(\frac{b}{a}\right)}\underset{(+)}{\left(\frac{c}{a}\right)} + \left(\frac{c}{a}\right)^2 + \underset{(+)}{\left(\frac{c}{a}\right)} > 0 \quad (5)$$

Therfore, eq^n (A) has positive (5) real roots. $\boxed{75}$

Let, $\frac{1}{\lambda} = y$ (5)

λ, root of (A),

Then, $\frac{1}{x} = y \implies$ (5) $x = \frac{1}{y}$

substituting, $x = \frac{1}{y}$ into (A) $\Rightarrow$

$$a^2\left(\frac{1}{y}\right)^2 + a(3b-2c)\left(\frac{1}{y}\right) + (2b-c)(b-c) + ac = 0$$

5

$$\left[(2b-c)(b-c) + ac\right]y^2 + a(3b-2c)y + a^2 = 0$$

$$\underline{\underline{(2b^2 - 3bc + c^2 + ac)y^2 + a(3b-2c)y + a^2 = 0}}$$ 5 20

b). (i) $A(k, 2)$ and $B(3, 4)$

$$AB = (k-3)^2 + (4-2)^2 = 64$$

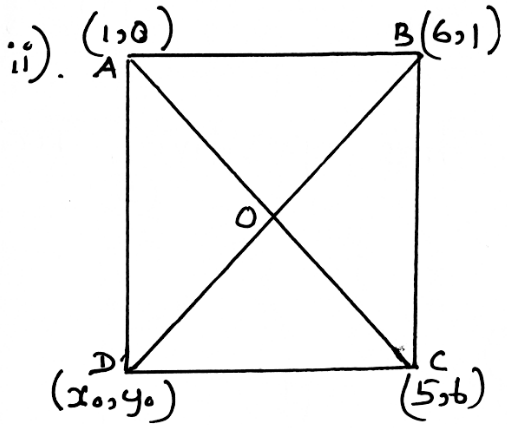
$$(k-3)^2 + 4 = 64$$

$$(k-3)^2 = 60$$

$$k-3 = \pm 2\sqrt{15}$$

$$\underline{\underline{k = 3 \pm 2\sqrt{15}}}$$ 15

ii).



$O \equiv (3, 3)$

$$3 = \frac{x_0 + 6}{2}, \quad 3 = \frac{y_0 + 1}{2}$$

$$x_0 = 0, \quad y_0 = 5$$

$\therefore \underline{\underline{D \equiv (0, 5)}}$ 15

$$\frac{4x^2}{(4x^2-1)} = \frac{(2x)^2}{(2x-1)(2x+1)}$$

$$x \longrightarrow 2x$$
$$a \longrightarrow 1 \quad (5)$$
$$b \longrightarrow -1$$

$$\frac{4x^2}{(4x^2-1)} = 1 + \frac{1}{2(2x-1)} + \frac{1}{(-2)(2x+1)} \quad (10)$$

$$\frac{4x^2}{(4x^2-1)} = 1 + \frac{1}{2(2x-1)} - \frac{1}{2(2x+1)} \quad (5) \qquad 50$$

e). $$\lim_{x\to\infty}\left(\sqrt{x^2+ax+a^2} - \sqrt{x^2+a^2}\right)$$

$$= \lim_{x\to\infty} \frac{\left(\sqrt{x^2+ax+a^2} - \sqrt{x^2+a^2}\right) \times \left(\sqrt{x^2+ax+a^2} + \sqrt{x^2+a^2}\right)}{\left(\sqrt{x^2+ax+a^2} + \sqrt{x^2+a^2}\right)} \quad (5)$$

$$= \lim_{x\to\infty} \frac{x^2+ax+a^2-x^2-a^2}{\sqrt{x^2+ax+a^2} + \sqrt{x^2+a^2}} \quad (5)$$

$$= \lim_{x\to\infty} \frac{ax}{\left(\sqrt{x^2+ax+a^2} + \sqrt{x^2+a^2}\right)} \quad (5)$$

$$= \lim_{x\to\infty} \frac{a}{\left(\sqrt{1+\frac{a}{x}+\frac{a^2}{x^2}} + \sqrt{1+\frac{a^2}{x^2}}\right)} \quad (5) \quad = \frac{a}{\sqrt{1}+\sqrt{1}} \quad (5)$$

$$(5) = \frac{a}{2} \qquad 30$$

16

a). $\sin(A+B) = \sin A\cos B + \cos A\sin B$ 5

5

$$\sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right) = \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \cos\frac{\pi}{6}\sin\frac{\pi}{4}$$

$$\sin\left(\frac{5\pi}{12}\right) = \frac{1}{2}\times\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}$$ 5

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$ 5

$$\underline{\underline{\sin\left(\frac{5\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}}}$$ 5

25

Hence,

$$\cos\left(\frac{5\pi}{6}\right) = 1 - 2\sin^2\left(\frac{5\pi}{12}\right)$$ 5

5 $$= 1 - 2\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)^2$$

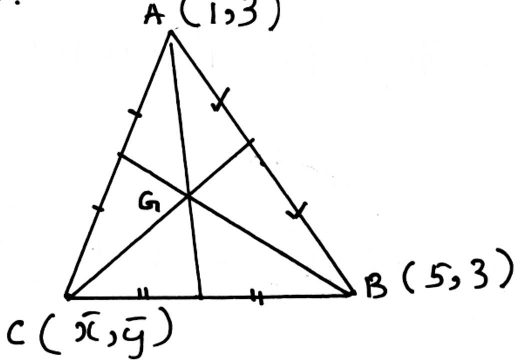
$$= 1 - \frac{1}{8}\left(8 + 2\sqrt{12}\right)$$ 5

$$= 1 - 1 - \frac{1}{4}\,2\sqrt{3}$$

20

$$\underline{\underline{\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}}}$$ 5

iii).



$$G \equiv \left(\frac{1+5+\bar{x}}{3} , \frac{3+3+\bar{y}}{3} \right)$$

$$\frac{10}{3} = \frac{6+\bar{x}}{3} \qquad 4 = \frac{6+\bar{y}}{3}$$

$$\bar{x} = 4 \qquad \bar{y} = 6$$

$\therefore$ $\underline{\underline{C(4,6)}}$ 25

55

15 (a). Proof - Factor Theorom 15

$$f(x) = x^4 + ax^3 + bx + c \equiv (x-1)(x+1)(x-2)(x+\lambda)$$ 5

$\underline{x=1}$ $\quad 1+a+b+c = 0$

$a+b+c = -1$ ——— (1) 5

$\underline{x=2}$ $\quad 16+8a+2b+c = 0$

$8a+2b+c = -16$ ——— (2) 5

$\underline{x=-1}$ $\quad 1-a-b+c = 0$

$c-a-b = -1$ ——— (3)

$\underline{\underline{a = \frac{-5}{2}}}$ $\quad \underline{\underline{b = \frac{5}{2}}}$ $\quad \underline{\underline{c = -1}}$ $\quad \underline{\underline{\lambda = -\frac{1}{2}}}$ 10

10

$\Rightarrow \left(x-\frac{1}{2}\right)$ 5 $\qquad f(x) = (x^4-1)(x+1)(x-2)\left(x-\frac{1}{2}\right)$

$$2f(x+1) = x^2+x-2$$

$$2\left\{x(x+2)(x-1)\left(x+\frac{1}{2}\right)\right\} = (x+2)(x-1)$$ 5

$$(x+2)(x-1)\left\{2x\left(x+\frac{1}{2}\right)-1\right\} = 0$$ 5

$$(x+2)(x-1)(2x^2+x-1) = 0$$

$$(x+2)(x-1)(2x-1)(x+1) = 0$$ 5

solutions are,

$x=-2 \qquad x=1 \qquad x=\frac{1}{2} \qquad x=-1$ 10 55

b). $$\frac{x^2}{(x-a)(x-b)} = p + \frac{q}{(x-a)} + \frac{r}{(x-b)}$$ 5

$$x^2 = p(x-a)(x-b) + q(x-b) + r(x-a)$$

$x^2 \longrightarrow \quad 1 = p$

$x \longrightarrow \quad 0 = -p(a+b) + q + r$ 10

$x^0 \longrightarrow \quad 0 = abp - bq - ar$

$p=1$ 5 $\qquad q = \frac{a^2}{a-b}$ 5 $\qquad r = \frac{b^2}{b-a}$ 5

$$\frac{x^2}{(x-a)(x-b)} = 1 + \frac{a^2}{(a-b)(x-a)} + \frac{b^2}{(b-a)(x-b)}$$

d). $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \pi/4$

Let, $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$ $\quad \beta = \tan^{-1}\left(\frac{1}{3}\right)$

$\tan\alpha = \frac{1}{2}$ $\quad \tan\beta = \frac{1}{3}$ 5

Then,

$$\alpha + \beta = \frac{\pi}{4}$$ 5

Prove that,

$$\tan(\alpha+\beta) = \tan\frac{\pi}{4} = 1$$ 5

L.H.S. $\tan(\alpha+\beta)$

$$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$ 5

$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}}$$ 5

$$= \frac{5}{5} = \underline{\underline{1}}$$ 5

$\therefore \underline{\underline{\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \pi/4}}$

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If,

b). $\alpha+\beta-\gamma = \pi$,

$\sin^2\alpha+\sin^2\beta-\sin^2\gamma = 2\sin\alpha\cos\gamma\sin\beta$

L.H.S $\sin^2\alpha+\sin^2\beta-\sin^2\gamma$

$= \sin^2\alpha + (\sin\beta - \sin\gamma)(\sin\beta+\sin\gamma)$ (5) (5)

$= \sin^2\alpha + 2\cos\left(\frac{\beta+\gamma}{2}\right)\sin\left(\frac{\beta-\gamma}{2}\right) 2\sin\left(\frac{\beta+\gamma}{2}\right)\cos\left(\frac{\beta-\gamma}{2}\right)$ (5) (5)

$= \sin^2\alpha + \sin(\beta+\gamma)\sin(\beta-\gamma)$ (5) $\beta-\gamma = \pi-\alpha$

$= \sin^2\alpha + \sin(\beta+\gamma)\sin\alpha$ (5)

$= \sin\alpha\,[\sin\alpha + \sin(\beta+\gamma)]$ (5)

$\alpha = \pi+\gamma-\beta$

$= \sin\alpha\,(\sin(\beta+\gamma) - \sin(\gamma-\beta))]$ (5)

$= \sin\alpha \cdot 2 \cdot \cos\gamma\sin\beta$

$= 2\sin\alpha\cos\gamma\sin\beta$ (5)

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c). $2\cos^2 x + \sqrt{3}\sin x + 1 = 0$

$2(1-\sin^2 x) + \sqrt{3}\sin x + 1 = 0$ (5)

$2\sin^2 x - \sqrt{3}\sin x - 3 = 0$ (5)

$(2\sin x + \sqrt{3})(\sin x - \sqrt{3}) = 0$

$\sin x = -\frac{\sqrt{3}}{2}$ (5) or $\sin x \neq \sqrt{3}$ (5)

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$\sin x = \sin(-\pi/3) \Rightarrow x = n\pi + (-1)^n(-\pi/3);\ n\in\mathbb{Z}$

(5) (5)

(17) To state - Sine Rule. (05)

a). $$\frac{a^2+b^2}{a^2+c^2} = \frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B}$$

L.H.S.

$$= \frac{a^2+b^2}{a^2+c^2}$$

From sine Rule,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{\lambda}$$ (5)

$A+B+C = \pi$ (5)

$$= \frac{\sin^2 A + \sin^2 B}{\sin^2 A + \sin^2 C}$$ (5)

$$= \frac{1-\cos 2A + 1-\cos 2B}{1-\cos 2A + 1-\cos 2C}$$ (10)

$$= \frac{2-(\cos 2A+\cos 2B)}{2-(\cos 2A+\cos 2C)}$$

$$= \frac{2-2\cos(A+B)\cos(A-B)}{2-2\cos(A+C)\cos(A-C)}$$ (10)

$$= \frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B}$$ (10)

$$\therefore \frac{a^2+b^2}{a^2+c^2} = \frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B}$$

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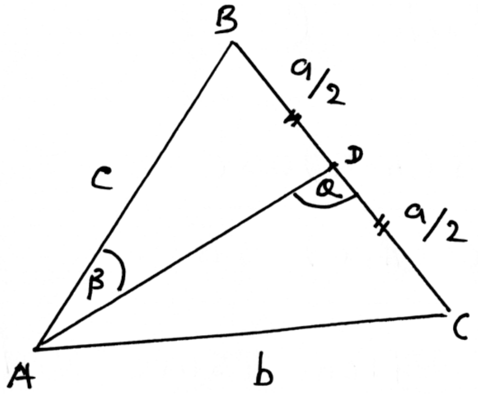
b)



(10)

ABD Δ, cosine Rule,

$$\cos A\hat{D}B = \frac{AD^2 + \left(\frac{a}{2}\right)^2 - c^2}{2AD.\left(\frac{a}{2}\right)} \quad (10) \quad \text{——} (1)$$

ADC Δ, cosine Rule,

$$\cos A\hat{D}C = \cos(\pi - A\hat{D}B) = -\cos A\hat{D}B \quad (5)$$

$$-\cos A\hat{D}B = \frac{AD^2 + \left(\frac{a}{2}\right)^2 - b^2}{2AD.\left(\frac{a}{2}\right)} \quad (10) \quad \text{——} (2)$$

From (1) and (2),

$$0 = 2AD^2 + 2\left(\frac{a}{2}\right)^2 - c^2 - b^2 \quad (5)$$

$$2AD^2 = b^2 + c^2 - \frac{a^2}{2}$$

$$AD^2 = \frac{2b^2 + 2c^2 - a^2}{4} \quad (5)$$

$$AD = \frac{\sqrt{2b^2 + 2c^2 - a^2}}{2} \quad (5)$$

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If $B\hat{A}D = \beta$,

$$\frac{\sin\beta}{a/2} = \frac{\sin B}{AD} \quad 10$$

$$\sin\beta = \frac{a\sin B \times 2}{2\sqrt{2b^2+2c^2-a^2}} \quad 10$$

$$\therefore \sin\beta = \frac{a\sin B}{\sqrt{2b^2+2c^2-a^2}} \quad 5$$

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If, $A\hat{D}C = \alpha$,

$$\frac{\sin\alpha}{b} = \frac{\sin C}{AD.} \quad 10$$

$$\sin\alpha = \frac{b\sin C \times 2}{\sqrt{2b^2+2c^2-a^2}} \quad 10$$

$$\sin\alpha = \frac{2b\sin C}{\sqrt{2b^2+2c^2-a^2}} \quad 5$$

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