අධ්‍යයන පොදු සහතික පත්‍ර (උ/පෙළ) විභාගය

12 ශ්‍රේණිය | **සංයුක්ත ගණිතය** | **පිළිතුරු පත්‍රය**

A කොටසේ එක් ප්‍රශ්නයකට ලකුණු 05 බැගින් ප්‍රශ්න 10 ට ලකුණු = 10 x 05 = 50
B කොටසේ එක් ප්‍රශ්නයකට ලකුණු 50 බැගින් ප්‍රශ්න 05 කට ලකුණු = 05 x 50 = 250
= 300

A කොටස

01. (a) $y = f(x) = \frac{1}{\sqrt{1 - x^2}}$

y තාත්වික වීමට $x - x^2 > 0$

$\Rightarrow x^2 - 1 < 0$

$(x - 1)(x + 1) < 0$

$\therefore$ f හි වසම $D_f = (-1, 1)$

$y = \frac{1}{\sqrt{1 - x^2}} \Rightarrow y^2 = \frac{1}{1 - x^2} \Rightarrow y^2 + x^2 = 1$

$x^2 = \frac{y^2 - 1}{y^2} = 1 - \frac{1}{y^2}$

තවද $y = \frac{1}{\sqrt{1 - x^2}} > 0$

y ට ඕනෑම අගයක් ගත හැකිය.

f හි පරාසය $R_f = \mathbb{R}^+$

(b) $y = f(x) = \frac{1}{(x - 2)(x - 3)}$

f හි වසම $D_f = \mathbb{R} - \{2, 3\}$

$yx^2 - 5yx + 6y = 1$

$yx^2 - 5y.x + (6y - 1) = 0$

x තාත්වික වීමට $\Delta = (-5y)^2 - 4y(6y - 1) \geq 0$

$y^2 + 4y = y(y + 4) \geq 0$

$y \leq -4$ හෝ $y \geq 0$

$y \in (-\infty, -4) \cup (0, \infty)$

f හි පරාසය $= (-\infty, -4) \cup (0, \infty)$

---

02. $\theta = 30^0 = 30 \times \frac{\pi}{180} = \frac{\pi}{6}$

$60^0 = 30 \times 2 = \frac{\pi}{3}$

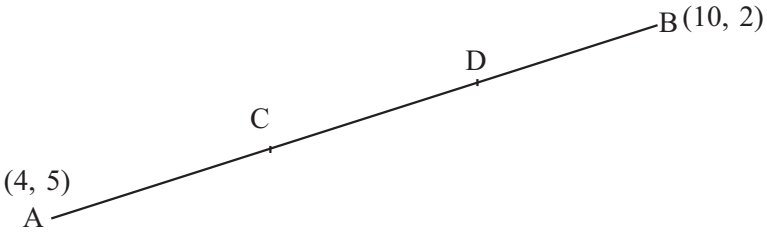
$120^0 = 30 \times 4 = \frac{2\pi}{3}$

ADC චාප දිග $= 7\sqrt{3} \times \frac{\pi}{3} = 3\frac{\sqrt{3}}{3}$

ABC චාප දිග $= 7 \times 2\frac{\pi}{3} = \frac{44}{3}$

$\therefore$ ABCD පරිමිතිය = ADC චාප දිග + ABC චාප දිග $= \frac{22}{3}(\sqrt{3} + 2)$ cm

03.



AC : CD = 1 : 2 නිසා

$$C = \left(\frac{2 \times 4 + 1 \times 10}{1 + 2}, \frac{2 \times 5 + 1 \times 2}{2 + 1}\right)$$

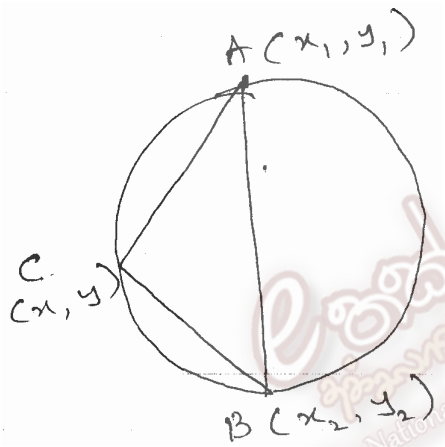
$$C \equiv \left(\frac{18}{3}, \frac{12}{3}\right) \equiv (6, 4)$$

AD : DC = 2 : 1 නිසා

$$D = \left(\frac{1 \times 4 + 2 \times 10}{1 + 2}, \frac{1 \times 5 + 2 \times 2}{1 + 2}\right)$$

$$D = (8, 3)$$

04.



AB විෂ්කම්භය වන වෘත්තය මත $C(x, y)$ ලක්ෂ්‍යය ඇත්නම්,

$APB = 90^0$

$AP^2 + BP^2 = AB^2$

$$[(x - x_1)^2 + (y - y_1)^2] + [(x - x_2)^2 + (y - y_2)^2] = [(x_1 - x_2)^2 + (y_1 - y_2)^2]$$

$$\therefore x^2 - 2xx_1 + x_1^2 + y^2 - 2yy_1 + y_1^2 + x^2 - 2xx_2 + x_2^2 + y^2 - 2yy_2$$

$$x^2 - xx_1 - xx_2 + x_1x_2 + y^2 - yy_1 - y_2 + y_1y_2 = 0$$

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

05.

$3^{2x} - 30 \times 3^x + 81 = 0$

$(3x)^2 - 30 \times 3^x + 81 = 0$

$(3^x - 27)(3^x - 3) = 0$

$3^x - 27 = 0$ හෝ $3^x - 3 = 0$

$3^x = 3^3$ හෝ $3^x - 3 = 0$

$x = 3$ හෝ $x = 1$

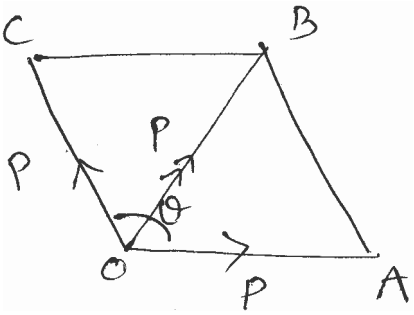
06.

$$\lim_{x \to 1} \left(\frac{x^{3/4} - 1}{x^{-2/3} - 1}\right) = \lim_{x \to 1} \left[\frac{\left(\frac{x^{3/4} - 1}{x - 1}\right)}{\left(\frac{x^{-2/3} - 1}{x - 1}\right)}\right]$$

$$= \left[\begin{matrix} \lim\limits_{x \to 1} \dfrac{x^{3/4} - 1^{3/4}}{x - 1} \\ \lim\limits_{x \to 1} \dfrac{x^{-2/3} - 1^{-2/3}}{x - 1} \end{matrix}\right]$$

$$= \frac{\left(\frac{3}{4}\right) 1^{\frac{3}{4} - 1}}{\left(-\frac{2}{3}\right) 1^{-\frac{2}{3} - 1}} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$$

07.



$P = P,\ Q = P,\ R = P$

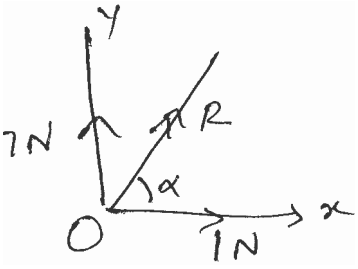
$R^2 = P^2 + Q^2 + 2PQ\ A\ Cos\ \theta$

$P^2 = P^2 + P^2 + 2P^2\ Cos\ \theta$

$-P^2 = 2P^2\ Cos\ \theta$

$Cos\ \theta = -\frac{1}{2} \quad \theta = 120^0$

08.



බල පද්ධතියේ සම්ප්‍රයුක්තය R නම්,

$R = \sqrt{1 + 7^2} = \sqrt{1 + 47} = \sqrt{50} = 5\sqrt{2}\ N$

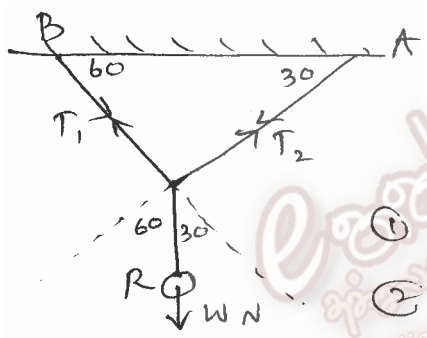
$\tan \alpha = \frac{7}{1}$

$\alpha = \tan^{-1}(7)$

සම්ප්‍රයුක්තය $5\sqrt{2}$ N වන අතර සම්ප්‍රයුක්තය OA සමඟ $\tan^{-1}$ 7 ක $\measuredangle$ ක් සාදයි.

$R = 1 + 7^2 = 1 + 47 = 50 = 5\ 2\ N$

09.



$\nearrow\ T_2 - W\ Cos\ 60 = 0$ ——①

$\nwarrow\ T_1 - W\ Sin\ 60 = 0$ ——②

①න් $T_2 = W\ Cos\ 60 = \frac{W}{2}$

②න් $T_1 = W\ Sin\ 60 = \frac{\sqrt{3}W}{2}$

10. $F_1 + F_2 + F_3 + F_4 = 0$

$a\underline{i} + 3\underline{j} + a\underline{i} + b\underline{j} + 5\underline{i} - 4\underline{j} - \underline{i} - 2\underline{j} = 0$

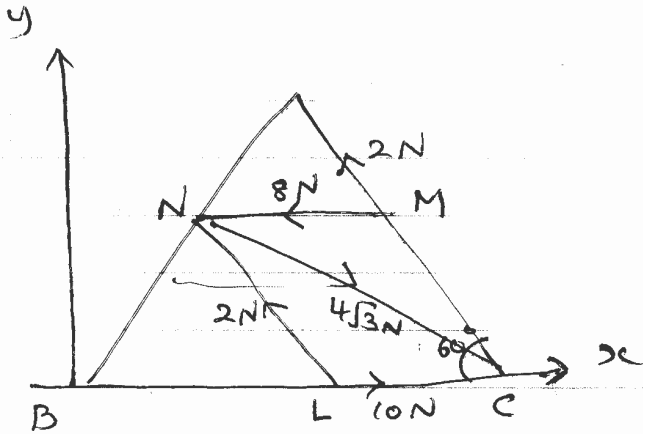
$(2a + 4)\underline{i} + (-3 + b)\underline{j} = 0$

$(2a + 4) = 0 \quad (-3 + b) = 0$

$a = -2\ ,\ b = 3$

B කොටස

11. (a)



$\overrightarrow{X} = 10 - 8 - 2\ Cos\ 60 - 2\ Cos\ 60 + 4\sqrt{3}\ Cos\ 30$

$= 2 - 4 \times \frac{1}{2} + 4\sqrt{3} \times \frac{\sqrt{3}}{2}$

$= 2 - 2 + 6 = 6$

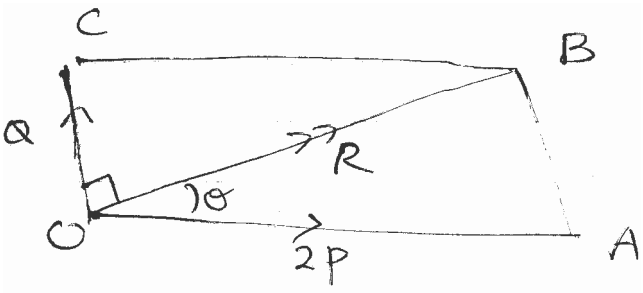
$Y\uparrow = 2\ Sin\ 60 + 2\ Sin\ 60 - 4\ 3\ P\ Sin\ 30$

$= 4\ Sin\ 60 - 4\ 3\ Sin\ 30$

$= 4 \times \frac{\sqrt{3}}{2} - 4\sqrt{3} \times \frac{1}{2} = 0$

BX දිශාවට 6N බලයකි

(b)



සම්ප්‍රයුක්තය සහ Q බලය අතර කෝණය θ යැයි ගනිමු.

OBC Δ යේ $= \frac{BC}{OC}$ Sin θ

$\frac{P}{Q}$ = Sin θ

P = Q Sin θ ——①

OAB Δ යේ $\frac{BA}{OA}$ = Sin θ නිසා $\frac{Q}{2P}$ = Sin θ

Q = 2P Sin θ ——②

① හා ② න් Q = 2θ Sin^2 θ

$\Rightarrow Sin^2 \theta = \frac{1}{2}$

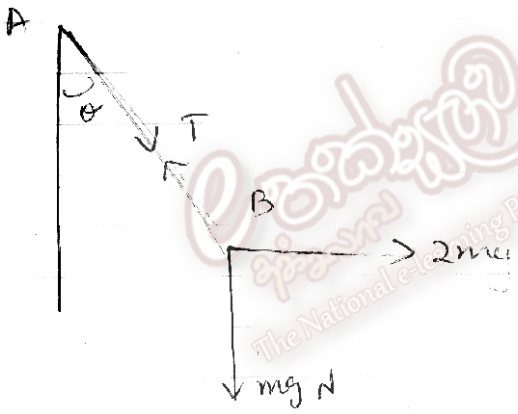
$Sin \theta = \pm \frac{1}{\sqrt{2}}$

θ සුළු කෝණයක් නිසා

θ = 45^0

බල දෙක අතර ∢ = $90^0 + \theta = 135^0$

12. (a)



↑ T Cos θ - mg = 0 ——①

← T Sin θ - 2mg = 0 ——②

T Cos θ = mg ——③

T Sin θ = 2mg ——④

$T^2 (Cos^2\theta + Sin^2\theta) = (mg)^2 + (2mg)^2$

$= 5(mg)^2$

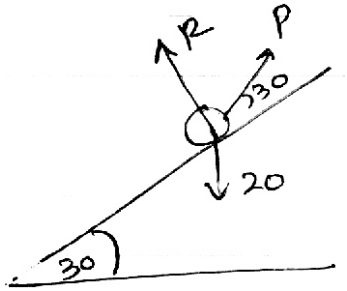
T = $\sqrt{5}$ mg

④/③ $\frac{T \text{ Sin } \theta}{T \text{ Sin } \theta} = \frac{2mg}{mg} = 2$

tan θ = 2 $\Rightarrow$ θ = $\tan^{-1}(2)$

ආතතිය T = $\sqrt{5}$ mg

(b)



↗ P Cos 30 - 20 Sin 30 = 0 ——①

↖ R - 20 Cos 30 + P Sin 30 = 0 ——②

① න් $P = \frac{20 \text{ Sin } 30}{\text{Cos } 30} = 20 \tan 30 = \frac{20\sqrt{3}\text{ N}}{3}$

② න් R = 20 Cos 30 - P Sin 30

$= 20 \times \frac{\sqrt{3}}{2} - \frac{20\sqrt{3}\text{ N}}{3} \times \frac{1}{2}$

$= 10\sqrt{3} - \frac{10\sqrt{3}}{3} = \frac{20\sqrt{3}\text{ N}}{3}$

13. $P(x)$ බහුපදය $(x - a)$ වලින් බෙදූ විට ශේෂය $P(a)$ වේ.

සාධනය : $P(x)$ බහුපදය $(x - a)$ වලින් බෙදූ විට ලබ්ධිය $\varnothing(x)$ ද ශේෂය R ද යැයි ගනිමු.

බෙදීමේ ඇල්ගෙරිතමයට අනුව,

$P(x) = (x - a)\ \varnothing(x) + R$

$x = a$ විට, $P(a) = (a - a)\ \varnothing(x) + R$

$P(a) = R$

$\therefore$ $P(x)$ බහු පදය $(x - a)$ වලින් බෙදූ විට ශේෂය R නම් $R = P(a)$

බෙදීමේ ඇල්ගෙරිතමයට අනුව,

$f(x) \equiv (x - 1)(x - 2)(x - 3)\ 9(x) + Ax^2 + Bx + C$

$x = 1$ විට $f(1) = 0 + A + B + C$

$1 = A + B + C$ ——— ①

$x = 2$ විට $f(2) = 4A + 2B + C$

$4A + 2B + C = 2$ ——— ②

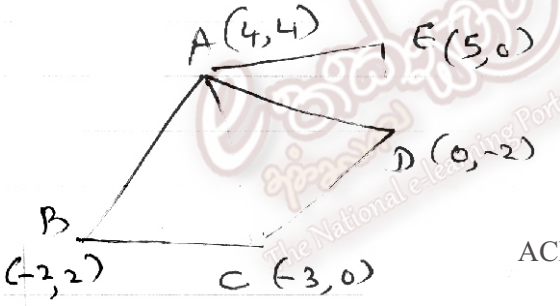
$x = 3$ විට $f(3) = 9A + 3B + C$

$9A + 3B + C = 3$ ——— ③

$A = 0,\ B = 1,\ C = 0$

$\therefore$ ශේෂය $Ax^2 + Bx + C = x$

14. (a)



ABC $\Delta = \frac{1}{2}\begin{vmatrix} -3 & 4 & -2 & -3 \\ 0 & 4 & 2 & 0 \end{vmatrix}$

$= \frac{1}{2}[-12 + 8 + 0 - (0 - 8 - 6)]$

$= 5$ වර්ග ඒකක

ACD Δ යේ වර්ගඵලය $= \frac{1}{2}\begin{vmatrix} -3 & 0 & 4 & -3 \\ 0 & -2 & 4 & 0 \end{vmatrix}$

$= \frac{1}{2} \times [26] = 13$ වර්ග ඒකක

ADE Δ යේ වර්ගඵලය $= \frac{1}{2}\begin{vmatrix} 0 & 5 & 4 & 0 \\ -2 & 0 & 4 & -2 \end{vmatrix} = \frac{1}{2} \times [22] = 11$ වර්ග ඒකක

$\therefore$ ABCDE පංචාස්‍රයේ වර්ගඵලය $= 5 + 13 + 11 = 29$ වර්ගඵල ඒකක

(b) A හා B හරහා යන රේඛාව $C \equiv (x, 0)$ ලක්ෂයේ දී x අක්ෂය කපන්නේ නම් හා C ලක්ෂ්‍යයෙන් AB බෙදෙන අනුපාතය $n : n$ යැයි සිතමු.

$$C \equiv \left[\frac{na + mc}{m + n},\ \frac{nb + md}{m + n}\right]$$

$C \equiv (x, 0)$ නිසා $\frac{nb + md}{m + n} = 0$

$\Rightarrow nb + md = 0$

$\Rightarrow md = -nb$

$\frac{m}{n} = \frac{-b}{d}$

$m : n = -b : d$

A හා B හරහා යන රේඛාව y අක්ෂයෙන් බෙදෙන ලක්ෂ්‍යය $D \equiv (0, y)$ යැයි ගනිමු. D ලක්ෂ්‍යයෙන් AB බෙදෙන අනුපාතය $\lambda : \mu$ යැයි ගනිමු.

$$D \equiv \left[\frac{\mu a + \lambda c}{\lambda + \mu} , \frac{\mu b + \lambda d}{\lambda + \mu} \right] \text{ නමුත් } D \equiv (0, y)$$

$$\therefore \frac{\mu a + \lambda c}{\lambda + \mu} = 0$$

$$\mu a + \lambda c \Rightarrow \lambda c = - \mu a$$

$$\frac{\lambda}{\mu} = \frac{-a}{c}$$

$$\Rightarrow \lambda : \mu = (-a) : c$$

AB රේඛාව y අක්ෂයෙන් බෙදෙන අනුපාතය $(-d) : c$

---

15. (a) (i) $\dfrac{1 - \text{Cos } 2A}{1 + \text{Cos } 2A} = \dfrac{1 - (1 - 2 \text{ Sin}^2 A)}{1 + (2 \text{ Cos}^2 A - 1)} = \dfrac{1 - 1 + 2 \text{ Sin}^2 A}{1 + 2 \text{ Cos}^2 A - 1} = \dfrac{2 \text{ Sin}^2 A}{2 \text{ Cos}^2 A}$

$$= \left(\frac{\text{Sin } A}{\text{Cos } A} \right)^2 = \tan^2 A$$

(ii) $$\frac{\text{Sec } 8A - 1}{\text{Sec } 4A - 1} = \frac{\dfrac{1}{\text{Cos } 8A} - 1}{\dfrac{1}{\text{Cos } 4A} - 1} = \frac{\dfrac{1 - \text{Cos } 8A}{\text{Cos } 8A}}{\dfrac{1 - \text{Cos } 4A}{\text{Cos } 4A}} = \frac{\dfrac{2 - \text{Sin}^2 4A}{\text{Cos } 8A}}{\dfrac{2 - \text{Sin}^2 2A}{\text{Cos } 4A}}$$

$$= \frac{\text{Sin}^2 4A}{\text{Cos } 8A} \times \frac{\text{Cos } 4A}{\text{Sin}^2 2A} = \frac{(\text{Sin } 4A \text{ . Cos } 4A) \text{ Sin } 4A}{\text{Cos } 8A (\text{Sin}^2 2A)}$$

$$= \frac{\text{Sin } 8A \text{ . Sin } 2A \text{ . Cos } 2A}{\text{Cos } 8A \text{ . Sin}^2 2A} = \frac{\text{Sin } 8A}{\text{Cos } 8A} = \frac{\text{Cos } 2A}{\text{Sin } 2A}$$

$$= \tan 8A \text{ . Cot } 2A = \frac{\tan 8A}{\text{Cot } 2A} = \text{RAS}$$

(b) $\text{Sin } 6\alpha = \text{Sin } [2 (3\alpha)] = 2 \text{ Sin } 3\alpha \text{ . Cos } 3\alpha$

$= 2[3 \text{ Sin } \alpha - 4 \text{ Sin}^3\alpha] [4 \text{ Cos}^3\alpha - 3 \text{ Cos } \alpha]$

$= 2[3 \text{ Sin } \alpha - 4 \text{ Sin}^3\alpha] \text{ Cos } \alpha [4 \text{ Cos}^3\alpha - 3]$

$= 2[3 \text{ Sin } \alpha - 4 \text{ Sin}^3\alpha] \text{ Cos } \alpha [4 (1 - \text{Sin}^2\alpha) - 3]$

$= 2\text{Cos } \alpha [3 \text{ Sin } \alpha - 4 \text{ Sin}^3\alpha] [1 - 4 \text{ Sin}^2\alpha]$

$$\frac{\text{Sin } 6\alpha}{2 \text{ Cos } \alpha} = (3 \text{ Sin } \alpha - 4 \text{ Sin}^3\alpha) (1 - 4 \text{ Sin}^3\alpha)$$

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16. (a) $\log \left(\frac{1}{3} + \frac{1}{4} \right) + 2 \log^2 + \log \frac{3}{7}$

$\log \frac{7}{12} + \log^2 + \log \frac{3}{7}$

$\log \left(\frac{7}{12} \text{ x } 2^2 \text{ x } \frac{3}{7} \right)$

$\log 1 = 0$

(b) $$\frac{1}{\log_{xy} xyz} + \frac{1}{\log_{yz} xyz} + \frac{1}{\log_{zx} xyz}$$

$$= \log_{xyz} xy + \log_{xyz} yz + \log_{xyz} zx$$

$$= \log_{xyz} x^2y^2z^2 = 2$$

(c) $a^{\log_a \frac{b}{c}} = y_1$ යැයි ගනිමු.

$\log_a y_1 = \log_a \frac{b}{c} \Rightarrow y_1 = \frac{b}{c} \Rightarrow a^{\log_a \left(\frac{b}{c}\right)} = \frac{b}{c}$

එසේම $b^{\log_a \frac{c}{d}} = \frac{c}{a}$ සහ $c^{\log_a \left(\frac{a}{b}\right)} = \frac{a}{b}$

$\therefore a^{\log_a \frac{b}{c}} \times b^{\log_b \frac{c}{a}} \times c^{\log_a \frac{c}{b}}$

$= \frac{b}{c} \times \frac{c}{a} \times \frac{a}{b} = 1$