අධ්‍යයන පොදු සහතික පත්‍ර (උ/පෙළ) විභාගය

12 ශ්‍රේණිය | **සංයුක්ත ගණිතය - I** | **පිළිතුරු පත්‍රය**

A කොටස

01. $a + b = x$ ——①
$b + c = y$ ——②
$a + c = z$ ——③
① - ② $a - c = x - y$, ① - ③ $b - c = x - z$, ② - ③ $b - a = y - z$

$$a^2 + b^2 + c^2 - ab - ac - bc = \frac{1}{2}[2a^2 + 2b^2 + 2c^2 - 2ab - 2ac - 2bc]$$
$$= \frac{1}{2}[a^2 - 2ab + b^2 + a^2 - 2ac + c^2 + b^2 - 2bc + c^2]$$
$$= \frac{1}{2}[(a-b)^2 + (a-c)^2 + (b-c)^2]$$
$$= \frac{1}{2}[(x-y)^2 + (y-z)^2 + (x-z)^2]$$

02. $3\,(1 + 11 + 111 + 1111 + ...)$
$\frac{3}{9}\ (9 + 99 + 999 + 9999 + ...)$
$= \frac{1}{3}\ [(10-1) + (100 - 1) + (1000 - 1) + ...]$
$= \frac{1}{3}\ [10 + 100 + 1000 + ... - (1 + 1 + ... n \text{ වරක්})]$
$= \frac{1}{3}\left[\frac{10\,(10^n - 1)}{10 - 1} - n\right]$
$= \frac{10}{27}\,(10^n - 1) - \frac{n}{3}$

03. $\lim\limits_{x \to 2} \left(\frac{\sqrt{1 + \sqrt{2 + x}} - \sqrt{3}}{x - 2}\right) \left(\frac{\sqrt{1 + \sqrt{2 + x}} + \sqrt{3}}{\sqrt{1 + \sqrt{2 + x}} + \sqrt{3}}\right)$

$$\lim_{x \to 2} \frac{1 + \sqrt{2 + x} - 3}{(x - 2)\,(\sqrt{1 + \sqrt{2 + x}} + \sqrt{3})} = \lim_{x \to 2} \frac{(\sqrt{2 + x} - 2)}{(x - 2)\,(\sqrt{1 + \sqrt{2 + x}} + \sqrt{3})} \times \frac{(\sqrt{2 + x} + 2)}{(\sqrt{2 + x} + 2)}$$

$$\lim_{x \to 2} \frac{2 + x - 4}{(x - 2)\,(\sqrt{1 + \sqrt{2 + x}} + \sqrt{3})\,(\sqrt{2 + x} + 2)}$$

$$\lim_{x \to 2} \frac{1}{(\sqrt{3} + \sqrt{3})\,(2 + 2)} = \frac{1}{8\sqrt{3}}$$

04. $3x^4 - 4x^3 - 14x^2 - 4x + 3 = 0$

$\div x^2 \quad 3x^2 - 4x - 14 - \frac{4}{x} + \frac{3}{x^2} = 0$

$3\left(x^2 + \frac{1}{x^2}\right) - 4\left(x + \frac{1}{x}\right) - 14 = 0$

$3(y^2 - 2) - 4y - 14 = 0$

$3y^2 - 4y - 20 = 0 \Rightarrow (3y - 10)(y + 2) = 0$

$y = \frac{10}{3}, \; y = -2$

$y = \frac{10}{3}$ විට $x = \frac{1}{3}, 3$

$y = -2$ විට $x = -1, -1$

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05. $p(x) = x^2 + 2\lambda x + 2\lambda - 1$

$\Delta = 4\lambda^2 - 4 \times 1 \times (2\lambda - 1)$

$= 4(\lambda^2 - 2\lambda + 1)$

$= 4(\lambda - 1)^2$

$\geq 0 \qquad \lambda \in IR \qquad p(x) = 0$ හි මූල තාත්වික වේ.

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06. $x^5 + ax^2 = (x + 1)(x - 2)(x - 3) \varnothing (x) + Ax + B$

$x = -1$ විට, $-1 + a = -A + B$ ——①

$\therefore -A + B - a = -1$ ——②

$x = 2$ විට $\quad 2A + B - 4a = 32$ ——③

$x = 3$ විට $\quad 3A + B - 9A = 243$

$a = -50, A = -39, B = -90$

ශේෂය $-39x - 90$

---

07. $1 + 2\log_{x+3} 7 = \log_7 (x + 3)$

$1 + 2 \times \frac{1}{\log_{x+3}} = \log_7 (x + 3)$

$y = \log_7 (x + 3)$ නම්, $1 + \frac{2}{y} = y$

$y^2 - y - 2 = 0$

$(y - 2)(y + 1) = 0$

$y = 2$ හෝ $y = -1$

$\log_7 (x + 3) = 2$ හෝ $\log_7 (x + 3) = -1$

$x + 3 = 7^2$ හෝ $x + 3 = \frac{1}{7}$

$x = 46$ හෝ $x = -\frac{20}{7}$

---

08. $1000N = 3456.456\ 456\ .....$

$= 3453 + 3.456\ 456\ 456$

$= 3453 + N$

$999N = 3453$

$N = \frac{3453}{999} = \frac{1151}{333}$

09. $y = 2x - 1$

$2x = y + 1$

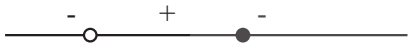
$x = \frac{1}{2}(y + 1)$

$\therefore f^{-1}(x) = \frac{1}{2}(x + 1)$, $x \in$ IR වේ.

10. $\frac{1}{x - 2} < 1$

$\frac{1}{x - 2} - 1 < 0$

$\frac{1 - (x - 2)}{x - 2} < 0 \Rightarrow \frac{3 - x}{x - 2} < 0$



$\therefore$ විසඳුම $x < 2$ හෝ $x > 3$ වේ.

B කොටස

(01) (i) $2\text{Cos}^2A = 1 + \text{Cos}\,2A$, $2\text{Sin}^2A = 1 - \text{Cos}2A$

$\tan A = \sqrt{\frac{1 - \text{Cos}\,2A}{1 + \text{Cos}\,2A}} = \sqrt{\frac{1 - \text{Cos}\,2A}{1 + \text{Cos}\,2A} \times \frac{1 - \text{Cos}\,2A}{1 - \text{Cos}\,2A}}$

$\tan A = \sqrt{\frac{(1 - \text{Cos}\,2A)^2}{1 - \text{Cos}^2 2A}}$

$= \sqrt{\frac{(1 - \text{Cos}\,2A)^2}{\text{Sin}^2 2A}} = \frac{1 - \text{Cos}\,2A}{\text{Sin}\,2A}$

$A = 22.5$ විට $\tan 22.5 = \frac{1 - \text{Cos}\,45}{\text{Sin}\,45} = \frac{1 - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \sqrt{2} - 1$

(ii) $3\text{Cos}\,x + 2\text{Sin}\,x = 3\left(\frac{1 - t^2}{1 + t^2}\right) + 2\frac{2t}{1 + t^2} = \frac{3 + 4t - 3t^2}{1 + t^2}$

$3\text{Cos}\,x + 2\text{Sin}\,x = 3$

$\frac{3 + 4t - 3t^2}{1 + t^2} = 3$

$6t^2 + 4t = 0$

$t(3t - 2) = 0 \Rightarrow t = 0$ හෝ $\frac{2}{3}$

$\tan \frac{x}{2} = 0$ හෝ $\frac{2}{3}$

$\frac{x}{2} = n\pi$ හෝ $\frac{x}{2} = n\pi + 0.588$

$x = 2n\pi$ හෝ $x = 2n\pi + 1.176$ $n \in z$

(iii) $r \operatorname{Cos} \alpha = 1$, $r \operatorname{Sin} \alpha = -\sqrt{3}$ ද වනසේ r හා α $(0 < \alpha < 2\pi)$ ගන්න.

$r = \sqrt{(-\sqrt{3})^2 + 1^2} = 2$ ද $\alpha = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\operatorname{Sin} x + \sqrt{3} \operatorname{Cos} x = 2 \operatorname{Cos} \alpha . \operatorname{Sin} x - 2 \operatorname{Sin} \alpha . \operatorname{Cos} x$

$= 2 \operatorname{Sin} (x - \frac{5\pi}{3})$

$\operatorname{Sin} x + \sqrt{3} \operatorname{Cos} x + 6 = 2 \operatorname{Sin} (x - \frac{5\pi}{3}) + 6$

එනම් , $4 \leq 2 \operatorname{Sin} (x - \frac{5\pi}{3}) \leq 8$

මේ අනුව $\operatorname{Sin} x + \sqrt{3} \operatorname{Cos} x + 6$ වැඩිම අගය 8 ද, අඩුම අගය 4 ද වේ.

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(02) (i) $f(x) = 2x^3 - 3x^2 - 5x + 8$

$f(-2) = (-10)$

$\therefore$ ශේෂය (-10)

(ii) $f(x) = x^3 - 7x^2 + 7x + 15$

$f(1) \neq 0$ $\therefore$ $x - 1$ සාධකයක් නොවේ.

$f(1) = 0$

$\therefore$ $(x + 1)$ සාධකයකි.

ඉතිරි සාධකය $x^2 - 8x + 15$ වේ.

$\therefore$ $x^3 - 7x^2 + 7x + 15 = (x + 1)(x^2 - 8x + 15)$

$= (x + 1)(x - 3)(x - 5)$

(iii) $\frac{4x^2 - 3x + 5}{(x+2)(x-1)^2} = \frac{A}{x + 2} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2}$

$4x^2 - 3x + 5 = A(x - 1)^2 + B(x + 2)(x - 1) + C(x + 2)$

$C = 2$, $A = 3$, $B = 1$

$\therefore$ $\frac{4x^2 - 3x + 5}{(x+2)(x-1)^2} = \frac{3}{x + 2} + \frac{1}{x - 1} + \frac{2}{(x - 1)^2}$

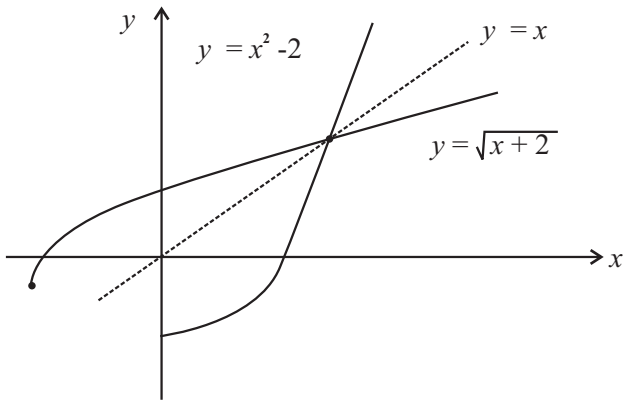
(iv) $y = x^2 - 2$

$x^2 = y + 2$

එනම් $x = \sqrt{y + 2}$ $(x \geq 0$ නිසා$)$

$\therefore$ $f^{-1}(x) = \sqrt{x + 2}$ $x \in IR$, $x \geq -2$

f හි වසම සීමාකර ඇත්තේ එය 1 - 1 ශ්‍රිතයේ කර ගැනීමටය.



(03) AB හි මධ්‍ය ලක්ෂ්‍යය $\left(\frac{3-1}{2}, \frac{2+4}{2}\right) = (1, 3)$

(i) AB හි අනුක්‍රමණය $= \frac{2-4}{3-(-1)} = \frac{-2}{4} = \frac{-1}{2}$

$\therefore$ AB ට $\perp$ රේඛාවක අනු $= 2$

AB ට ලම්භ සමච්ඡේදකය යනු එහි මධ්‍ය ලක්ෂ්‍ය හරහා AB ට ලම්භව ඇඳි රේඛාව වේ.

එබැවින් AB හි ලම්භ සමච්ඡේදකයේ සමීකරණය,

$y - 3 = 2(x - 1) \Rightarrow 2x - y + 1 = 0$

P (x, y) අවශ්‍ය පථයේ විචල්‍ය ලක්ෂයක් නම්,

$PA = PB \Rightarrow PA^2 = PB^2$

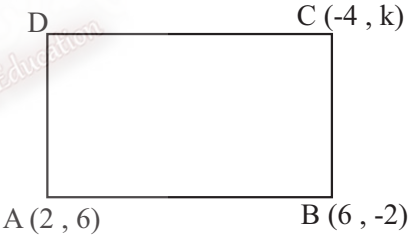
$(x - 3)^2 + (y - 2)^2 = (x + 1)^2 + (y - 4)^2$

$-8x + 4y - 4 = 0$

$2x - y + 1 = 0$

මේ අනුව A හා B ලක්ෂ්‍යවලින් සමදුරට පිහිටි ලක්ෂ්‍යයේ පථය AB රේඛා ඛණ්ඩයේ ලම්බ සමච්ඡේදකය වේ.

(ii) (a) AB හි අනුක්‍රමණය $\frac{6-(-2)}{2-6} = -2$



(b) AB පාදය BC පාදයට $\perp$ නිසා BC අනුක්‍රමණය $\frac{1}{2}$ වේ.

$\therefore \frac{k-(-2)}{-4-6} = \frac{1}{2} \Rightarrow k + 2 = -5k$

$k = -7$

(c) AD පාදයේ සමීකරණය

$y - 6 = \frac{1}{2}(x - 2) \Rightarrow 2y - x = 10$

CD පාදයේ සමීකරණය $y + 7 = -2(x + 4)$

$y + 2x = -15$

(d) $2y - x = 10$ හා $y + 2x = -15$ විසඳීමෙන්,

$x = -8$, $y = 1$

$\therefore$ D $\equiv (-8, 1)$

---

(04) (i) $\lim_{x \to 8} \frac{\sqrt{x+1} - 3}{x - 8} = \lim_{x \to 8} \frac{(\sqrt{x+1} - 3)(\sqrt{x+1} + 3)}{(x - 8)(\sqrt{x+1} + 3)}$

$= \lim_{x \to 8} \frac{x + 1 - 9}{(x - 8)(\sqrt{x+1} + 3)} = \frac{1}{6}$

(ii) $\lim_{x \to -2} \frac{x^5 + 2^5}{x + 2} = \lim_{x \to -8} \frac{x^5 - 2^5}{x - 2} = 5(-2)^4 = 80$

(iii) $|x^2 + 2x - 3| \leq 3$ අසමානතාව

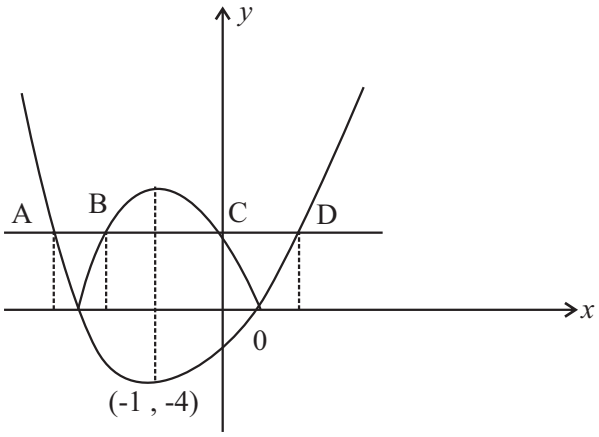
$y = |x^2 + 2x - 3|$ හා $y = 3$ ප්‍රස්තාර සලකමු.

$y = x^2 + 2x - 3 = (x - 1)(x + 3)$

$y = 0 \Rightarrow x = 1 \quad x = -3$

$y = (x + 1)^2 - 4 > -4$

අ.ල. $(-1, 4)$



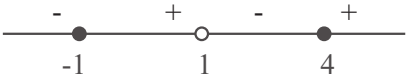
(iv) $\frac{2}{x-1} \geq 2 - x$

$\frac{2}{x-1} - (2 - x) \geq 0$

$\frac{2 - (2 - x)(x - 1)}{x - 1} \geq 0$

$\frac{2 - (-x^2 - 3x - 2)}{x - 1} \geq 0$

$\frac{(x + 1)(x - 4)}{x - 1} \geq 0$



විසඳුම $-1 , x \leqslant 1$ හෝ $x \geq 4$

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(05) (i) (a) $\alpha + \beta = \frac{4}{3} \qquad -\frac{(k+1)}{k} = -\frac{4}{3} \qquad 3k + 3 = 4k \Rightarrow k = 3$

(b) $k = 3$ නිසා $\quad 3x^2 + 4x - 5 = 0 \quad \alpha\beta = -\frac{5}{3}$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-\frac{4}{3}}{-\frac{5}{3}} = \frac{4}{5}$$

(c) $\alpha^2 + \beta^2 = \alpha^2 + 2\alpha\beta + \beta^2 - 2\alpha\beta = (\alpha + \beta)^2 - 2\alpha\beta$

$$= \left(-\frac{4}{3}\right)^2 - 2\left(-\frac{5}{3}\right)$$

$$\alpha^2 + \beta^2 = \frac{16}{9} + \frac{10}{3} = \frac{46}{9}$$

$$\alpha^2\beta^2 = (\alpha\beta)^2 = \left(-\frac{5}{3}\right)^2 = \frac{25}{9}$$

$\therefore \alpha^2$ හා β^2 මූල ඇති සමීකරණය වන්නේ,

$x^2 - \frac{46}{9}x + \frac{25}{9} = 0 \qquad 9x^2 - 46x + 25 = 0$

(ii) $y = \log_5 x$ විට $\log_x 5 = \frac{1}{\log_5 x} = \frac{1}{y}$

$y - \frac{4}{y} - 3 = 0$

$y^2 - 3y - 4 = 0$

$(y - 4)(y + 1) = 0$

$y = 4$ හෝ $y = -1$

$\log_5 x = 4$ හෝ $\log_5 x = -1$

$x = 5^4 = 625 \ , \ x = 5^{-1} = \frac{1}{5}$

(06) (i) $\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-\sqrt{3}} = \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+\sqrt{3}}$

$= \frac{18+3\sqrt{6}+6\sqrt{6}+6}{18-3} = \frac{3(8+3\sqrt{6})}{15}$

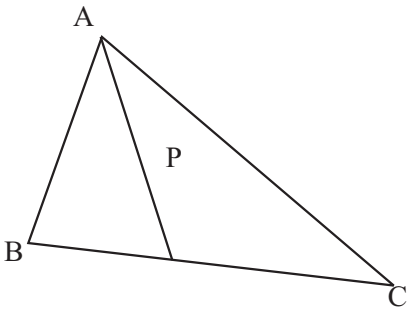
$= \frac{8+3\sqrt{6}}{5}$

(ii) (a) BC පාදයේ අනුක්‍රමණය $\frac{y_2-y_3}{x_2-x_3}$

$\therefore$ BC හි සමීකරණය වන්නේ,

$y-y_2 = \frac{y_2-y_3}{x_2-x_3}(x-x_3)$

$(x-x_2)(y_2-y_3) - (x_2-x_3)(y-y_2) = 0$



(b) A සිට BC පාදයට ඇති ලම්භ දුර P නම්,

$P = \frac{[(x_1-x_2)(y_2-y_3) - (x_2-x_3)(y_1-y_2)]}{\sqrt{(x_2-x_3)^2+(y_2-y_3)^2}}$

(c) BC දිග $= \sqrt{(x_2-x_3)^2+(y_2-y_3)^2}$

ABC Δ යේ වර්ගඵලය $\frac{1}{2}$ BC . P

$= \frac{1}{2}[(x_1-x_2)(y_2-y_3) - (x_2-x_3)(y_1-y_2)]$

$= \frac{1}{2}[(x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_1 - x_1y_3]$

$= \frac{1}{2}[3(-6) - 1(-2) + (-2)5 - (-6)4 + 4(1) - 3 \times 5]$

$= \frac{1}{2}[-18 + 2 - 10 + 24 + 4 - 15] = \frac{1}{2}[-13] =$ ව.ඒ. 6.5

සංයුක්ත ගණිතය - II

A කොටස

A කොටසේ ප්‍රශ්න 10 ට ලකුණු 25 බැගින් ද, B කොටසේ ප්‍රශ්නයකට ලකුණු 150 බැගින් ද හිමිවේ. $10 \times 25 = 250$

$5 \times 150 = 750$

$\underline{1000}$

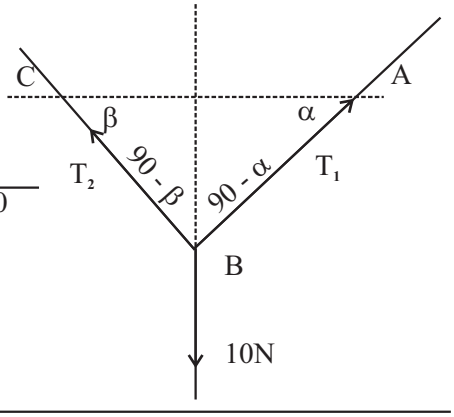
01. AC = 10m

AB = 8m $\quad A\hat{B}C = 90^0$

BC = 6m

$\cos\beta = \frac{3}{5} \quad \cos\alpha = \frac{4}{5}$

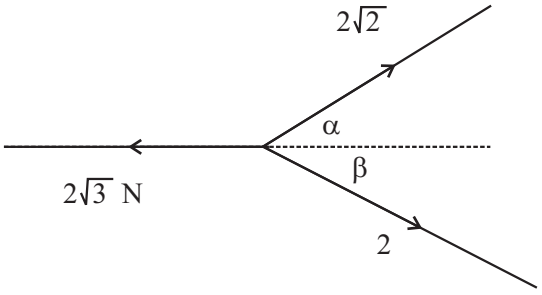
ලාමී ප්‍රමේයයෙන් $= \frac{T_1}{\sin(90+\beta)} = \frac{T_2}{\sin(90+\alpha)} = \frac{10}{\sin 90}$

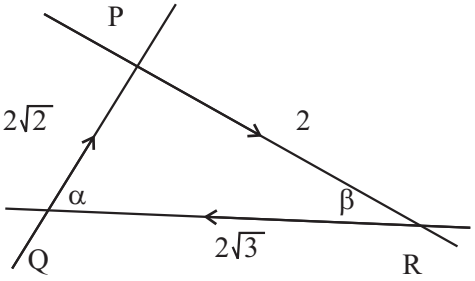


$\frac{T_1}{\cos\beta} = \frac{T_2}{\cos\alpha} = 10$

$\underline{T_1 = 6N, \; T_2 = 8N}$

02.





$PQ = 2\sqrt{2}$, $PR = 2$, $QR = 2\sqrt{3}$, $\hat{QPR} = 90^0$

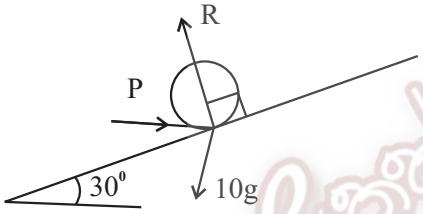
PQR Δ ට Sin සූත්‍රයෙන්,

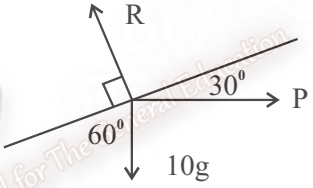
$$\frac{2\sqrt{3}}{\text{Sin } 90} - \frac{2}{\text{Sin } \alpha} = \frac{2\sqrt{2}}{\text{Sin } \beta}$$

$$\text{Sin } \alpha = \frac{1}{\sqrt{3}} \qquad \alpha = \text{Sin}^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\text{Sin } \beta = \frac{2\sqrt{2}}{2\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \qquad \beta = \text{Sin}^{-1}\sqrt{\frac{2}{3}}$$

03.





ලාමි ප්‍රමේයයෙන් $= \frac{R}{\text{Sin } 90} = \frac{P}{\text{Sin } (90 + 60)} = \frac{10g}{\text{Sin } (90 + 30)}$

$$R = \frac{P}{\text{Cos } 60} = \frac{10g}{\text{Cos } 30} \Rightarrow P = \frac{10g}{\text{Cos } 30} \cdot \text{Cos } 60$$

$$P = \frac{10g}{\sqrt{3}}$$

04. $u = 0$

$S = 100m$, $V = 20ms^{-1}$, $t = ?$, $a = ?$

$v^2 = u^2 + 2as$

$20^2 = 0 + 2 \times a \times 100$

$a = 2\ ms^{-2}$

$s = \frac{1}{2}(u + v)\,t$

$100 = \frac{1}{2}(0 + 20)t$

$t = 10\ s$

05. තිරස් සංරචකය $8 \text{ Cos } 30 = 4\sqrt{3}$

සිරස් සංරචකය $8 \text{ Sin } 30 = 4$

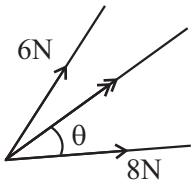
$\therefore V = (4\sqrt{3}\,\underline{i} + 4\,\underline{j})\ ms^{-1}$

06. (a) $P = 8N$, $\theta = 6N$, $\alpha = 90$

$R^2 = 8^2 + 6^2 + 2 \times 8 \times 6 \ \text{Cos } 90 = 100$

$R = 10$

$\tan \theta = \dfrac{6 \text{ Sin } 90}{8 + 6 \text{ Cos } 90} = \dfrac{6}{8} \qquad \theta = \tan^{-1}\left(\dfrac{3}{4}\right)$



(b) $P = 8N$, $\theta = 6$, $\alpha = 90$

$R^2 = 8^2 + 6^2 + 2 \times 8 \times 6 \ \text{Cos } 60 = 100 + 40$

$R = 2\sqrt{37}$

$\tan \theta = \dfrac{6 \text{ Sin } 60}{8 + 6 \text{ Cos } 60} = \dfrac{3\sqrt{3}}{8+3} = \dfrac{3\sqrt{3}}{11}$

07. $\vec{x} = 1 - 5\ 8\sqrt{2}\ \text{Cos } 45 = -12$

$y_{\uparrow} = 3 - 7 + 8\sqrt{2}\ \text{Sin } 45 = 4$

සම්ප්‍රයුක්ත බලයේ විශාලත්වය

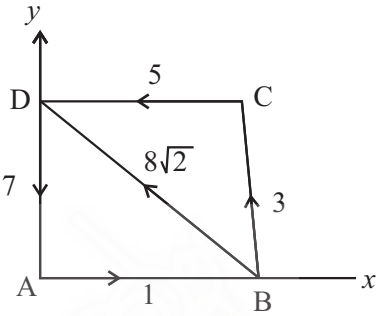
$= \sqrt{12^2 + 4^2} = 4\sqrt{10}$

$\overrightarrow{AG} = 3a + 5a + 8\sqrt{2} \times a \ \text{Cos } 45 = 16a$

P (x , y) සම්ප්‍රයුක්තයේ ක්‍රියා රේඛාව මත ලක්ෂ්‍යයක් ගනිමු.

P වටා $\circlearrowleft$ $-12y - 4x + 16a = 0$

$x + 3y - 4a = 0$

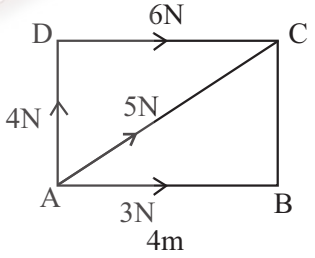


08. A) සම්ප්‍රයුක්ත සූර්ණය $= 6 \times 3 = 18$ Nm

B) සම්ප්‍රයුක්ත $= (6 \times 3) + (4 \times 4) + 5 \times 4 \ \text{Sin } B\hat{A}C$

$= 46$ Nm

C) $= (4 \times 4) - 3 \times 3 = 7$ Nm



09. $AC = \underline{C}$ නම් $CB = 2\underline{C}$

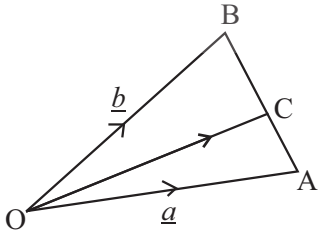
OAC Δ යේ $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \underline{a} + \underline{C}$

OBC Δ යේ $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \underline{b} - 2\underline{C}$

$\underline{a} + \underline{C} = b - 2\underline{C}$

$\underline{C} = \dfrac{\underline{b} - \underline{a}}{3}$

$OC = a + \dfrac{\underline{b} - \underline{a}}{3} = \dfrac{2\underline{a} + \underline{b}}{3}$



10. A $R\sqrt{4a^2 - a^2} = wa \ \text{Cos } 60$

$R = \dfrac{1}{2\sqrt{3}}\ w$

$\rightarrow R - x = 0$

$x = R = \dfrac{1}{2\sqrt{3}}\ w$

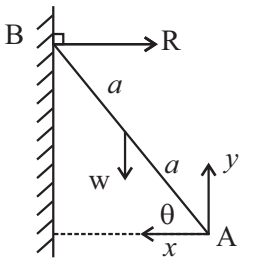
$\uparrow y = w$

$\therefore$ B හි ප්‍රතික්‍රියාව $\dfrac{1}{2\sqrt{3}}\ w$

A හි ප්‍රතික්‍රියාව $\sqrt{x^2 + y^2} = \sqrt{\dfrac{w^2}{12} + w^2} = \dfrac{\sqrt{39}}{6} w$

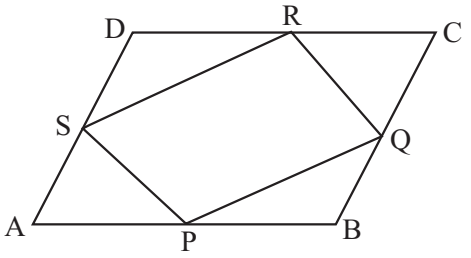
$\theta = \tan^{-1}\dfrac{y}{x}$

$= \tan^{-1}(2\sqrt{3})$



B කොටස

01. $\overrightarrow{AB} = \underline{a}$, $\overrightarrow{BC} = \underline{b}$ නම්



$\overrightarrow{CD} = -\underline{a}$, $\overrightarrow{DA} = -b$

$\overrightarrow{AP} = k\underline{a}$, $\overrightarrow{BQ} = k\underline{b}$, $\overrightarrow{CR} = -k\underline{a}$, $\overrightarrow{DS} = kb$

$\overrightarrow{PB} = \overrightarrow{AB} - \overrightarrow{AP} = \underline{a} - k\underline{a} = (1 - k)\,\underline{a}$

එලෙසම $\overrightarrow{DR} = (1 - k)\,\underline{a}$

$\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BQ}$

$= (1 - k)\,\underline{a} + k\underline{b}$ සහ $\overrightarrow{SR} = \overrightarrow{SD} + \overrightarrow{DR} = k\underline{b} + (1 - k)\,\underline{a}$

$\therefore \overrightarrow{PQ} = \overrightarrow{SR}$

PQRS සමාන්තරාස්‍රයකි.

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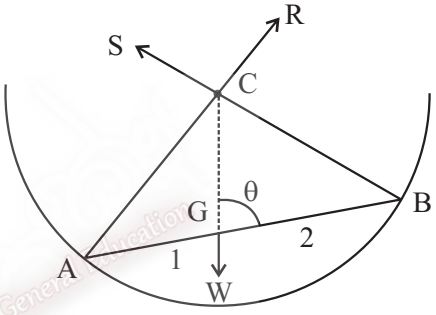
02. ACB ත්‍රිකෝණ සඳහා Cot ප්‍රමේයයන්



$(1 + 2)\,\text{Cot}\,\theta = 2\,\text{Cot A} - \text{Cot B}$

$3\,\text{Cot}\,\theta = 2\,\text{Cot}\,60 - \text{Cot}\,60$

$\frac{3}{\tan\theta} = \frac{1}{\tan 60} = \frac{1}{\sqrt{3}}$

$\tan\theta = 3\sqrt{3}$

$\theta = \tan^{-1}(3\sqrt{3})$

---

03. $a = \tan\theta$

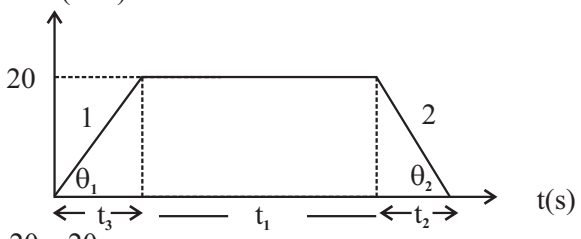
$1 = \frac{v}{60}$

$v = 60\ \text{ms}^{-1}$

$\frac{60}{t_1} = 2$

$t_1 = 30$

AB අතර දුර $= \frac{90 \times 60}{2} = 2700$ m



$\frac{20 \times 20}{2}$ = ත්වරණයෙන් ගිය දුර

ත්වරණයෙන් ගිය දුර $= 200$m

මන්ධයෙන් ගිය දුර $= \frac{20 \times 10}{2} = 100$ m

ඒකාකාර ප්‍රවේගයෙන් ගිය දුර $= (2700 - 300)$m

$= 2400$ m

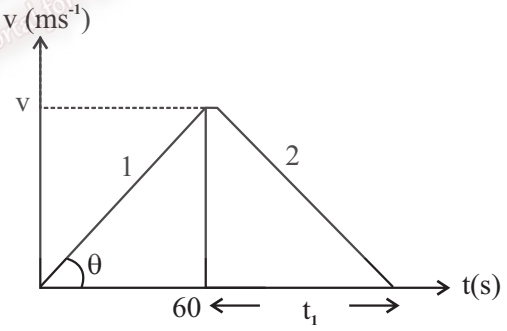
$20\,t_1 = 2400$

$t_1 = 120$

ප්‍රවේගයෙන් ගිය කාලය $= 120$ s

ගමනට ගතවන කාලය $= 120 + 10 + 20$

$= 150$ s

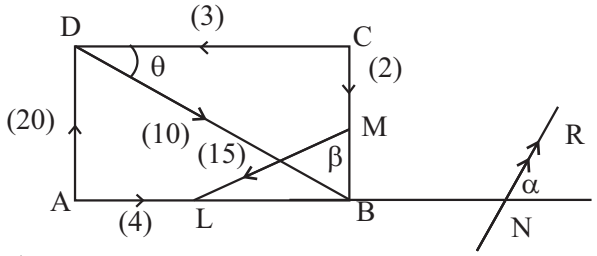


$a_1 = \tan\theta_1$ $\quad$ $a_2 = \tan\theta_2$

$1 = \frac{20}{t_3}$ $\quad$ $2 = \frac{20}{t_2}$

$t_3 = 20$ $\quad$ $t_2 = 10$

04.

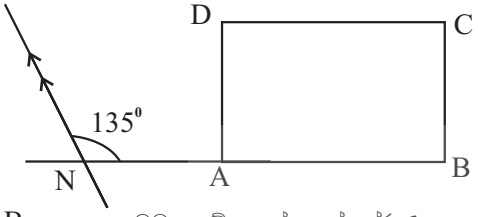


$\overrightarrow{x}$ = 4 + 10 Cos θ - 3 - 15 Sin β = (-3)

$y\uparrow$ = 20 - 2 - 10 Sin θ - 15 Cos β = 3

$R^2 = \sqrt{x^2 + y^2} \Rightarrow R = 3\sqrt{2}\ N$

$\alpha = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(-1) = 135^0$



B ලක්ෂ්‍යය වටා සම්ප්‍රයුක්තයේ සූර්ණය = B වටා බල පද්ධතියේ සූර්ණවල එකතුව

$R \cdot \text{Sin}\alpha \cdot BN = 3 \times 6 - 20 \times 8 + 15\ \text{Sin}\beta \times 3$

$3\ BN = 18 - 160 + 36$

$BN = \frac{-106}{3} = -35\frac{1}{3}$ cm

$\therefore BN = 35\frac{1}{3} - 8 = 27\frac{1}{3}$ cm

05. $AG = GB = \frac{a}{2}$, $BC = l$

ABC ත්‍රිකෝණයට Sin සූත්‍රය යෙදීමෙන්,

$\frac{\text{Sin}\,\alpha}{AB} = \frac{\text{Sin}\,(\pi - \theta)}{BC}$

$BC\ \text{Sin}\,\alpha = AB\ \text{Sin}\,\theta$

$l\ \text{Sin}\,\alpha = a\ \text{Sin}\,\theta$ ——①

AOB Δ ට Cot සූත්‍රය යෙදීමෙන්,

$(1 + 1)\ \text{Cot}\,\theta = \text{Cot}\,\alpha - \text{Cot}\,90$

$2\ \text{Cot}\,\theta = \text{Cot}\,\alpha$ ——②

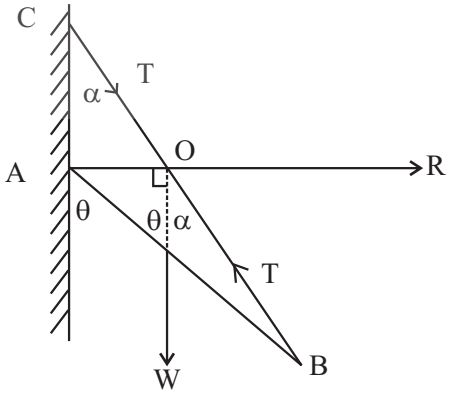
$4\ \text{Cot}^2\theta = \text{Cosec}^2\alpha - 1$

$= \frac{1}{\text{Sin}^2\alpha} - 1$

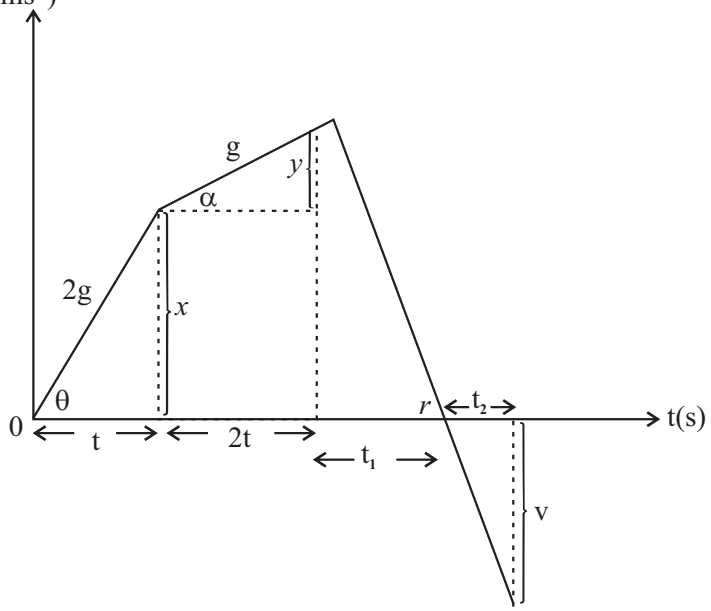
$= \frac{l^2}{a^2\ \text{Sin}^2\theta} - 1$

$\frac{4\ \text{Cos}^2\theta}{\text{Sin}^2\alpha} = \frac{l^2 - a^2\ \text{Sin}^2\theta}{a^2\ \text{Sin}^2\theta}$

$\text{Cos}^2\theta = \frac{l^2 - a^2}{3a^2}$



06. v (ms^{-1})



$\tan\theta = \frac{x}{t}$ $\tan\alpha = \frac{y}{2t}$, $\tan\alpha = g$

$\tan\theta = 2g$ $\underline{\underline{y = 2gt}}$

$\underline{\underline{x = 2gt}}$

$\tan r = \frac{4gt}{t_1}$

$g = \frac{4gt}{t_1}$

$t_1 = \frac{4gt}{g} = 4t$

උපරිම වේ $= 2t \times 2gt + \frac{2gt^2}{2} + \frac{2gt^2 \times 2t}{2} + \frac{4gt \times 4t}{2}$

$= 15\,gt^2$

$15gt^2 = \frac{vt_2}{2}$

$t_2 = \frac{30\,gt^2}{v}$

$g = \frac{v^2}{30gt^2}$

$v^2 = 30g^2t^2$

$v = gt\sqrt{30}$