29

Probability

By studying this lesson you will be able to,

- identify what the fraction of success of an outcome of a random experiment is,
- identify what the experimental probability of an event is, and
- identify what the theoretical probability of an event is.

29.1 Likelihood of an event occurring

Let us consider some events that occur in the environment. "The rising of the sun from the east" is an event that definitely occurs.

"The appearance of a full moon on a new moon day" is an event that definitely does not occur.

Let us consider the event of "a coin landing heads up when it is flipped". We cannot say definitely which of the two events "landing heads up" and "landing tails up" will occur. Therefore this is a **random event.**

You learnt in grade 7 that events can be classified into three groups. They are;

- Events which definitely occur
- Events which definitely do not occur
- Random events

Consider flipping a coin.

- The experiment is flipping the coin and observing the side that lands up.
- The outcomes of this experiment are; "the coin landing heads up" and "the coin landing tails up".
- If the coin is a fair (unbiased) coin, then there is an equal likelihood of each of these two outcomes occurring.

- The likelihood of an event which definitely does not occur is 0.
- The likelihood of an event which definitely occurs is 1.
- The likelihood of a random event occurring takes a value between 0 and 1, based on its tendency to occur.

Accordingly,

the likelihood of the sun rising from the west is 0,

the likelihood of the sun rising from the east is 1 and

the likelihood of a coin landing heads up when it is flipped takes a value between 0 and 1.

When a fair coin is flipped, there is an equal likelihood of it landing heads up and not landing heads up. Therefore we consider the likelihood of it landing heads up as $\frac{1}{2}$ and not landing heads up (landing tails up) as $\frac{1}{2}$.

- If there is an equal chance of an event occurring and of not occurring, then the likelihood of each is $\frac{1}{2}$.
- If the chance of an event occurring is greater than the chance of it not occurring, then the likelihood of that event occurring is a value between $\frac{1}{2}$ and 1.
- If the chance of an event not occurring is greater than the chance of it occurring, then the likelihood of that event occurring is a value between 0 and $\frac{1}{2}$.
- If p is the likelihood of a random event occurring, then the likelihood of it not occurring is $1 - p$.

When rolling an unbiased die with its faces marked with the numbers 1 to 6 (one number on each face), since there is an equal chance of each of the numbers from 1 to 6 showing up, the likelihood of 1 showing up is taken as $\frac{1}{6}$. Accordingly, the likelihood of 1 not showing up is $1 - \frac{1}{6} = \frac{5}{6}$.

Exercise 29.1

(1) Write three events which definitely occur.

(2) Write three events which definitely do not occur.

(3) Write three random events.

(4) An unbiased regular tetrahedral die with its faces numbered 1, 2, 3, 4 is tossed. Write the outcomes of the experiment of observing the number on the face which lands down.

(5) Complete the table given below.

	Event	Likelihood or the interval in which the likelihood lies (0, 1, $\frac{1}{2}$, between 0 and $\frac{1}{2}$, between $\frac{1}{2}$ and 1)
1	A fruit dropping from a tree landing on the ground	
2	The sun rising from the east	.....
3	If today is Monday, tomorrow being Wednesday	.....
4	A bead drawn from a bag containing 10 red beads and 2 blue beads of equal size and shape, being a red bead	.....
5	A face marked 1 showing up when a fair die with the numbers 1, 1, 1, 2, 2, 2 marked on its six faces is rolled	.....
6	Winning the toss in a match	.....
7	Getting a value greater than 2 when a fair die marked 1 to 6 is rolled	.....
8	The sum of two odd numbers being an even number	.....
9	The birthday of a child picked randomly from your class, falling on January 2	.....
10	A person passing away on a Monday	.....

29.2 Experimental Probability

• Random experiment

Let us consider again the event of a coin landing tails up when flipped. When the coin is being flipped, we cannot say with certainty whether it would land heads up or tails up. You have learnt that such an event is called a random event.

The experiment is flipping a coin and observing the side that lands up.

The outcomes of this experiment are "the coin landing heads up" and "the coin landing tails up".

An experiment of which the possible outcomes are known, but the actual outcome cannot be stated with certainty before the experiment is conducted is called a **random experiment**.

A random experiment and its possible outcomes are given in the following table.

Random Experiment	Possible Outcomes
Rolling a die with its faces numbered 1 to 6, and observing the number which shows up	1 showing up, 2 showing up 3 showing up, 4 showing up 5 showing up, 6 showing up

A random experiment has the following common features.

- Can be repeated any number of times under the same conditions
- The actual outcome cannot be stated with certainty before the experiment is conducted
- All the possible outcomes of the experiment are known before the experiment is conducted

• Fraction of Success (Relative frequency)

A Rs. 2 coin was flipped 20 times and the side that landed up was observed. The following are the outcomes.

The coin landed heads up 11 times.
The coin landed tails up 9 times.

The fraction of success of the coin landing heads up is

$$\frac{\text{the number of times the coin landed heads up}}{\text{the number of time the coin was flipped}}$$

$\therefore$ The fraction of success of the coin landing heads up $= \frac{11}{20}$

The fraction of success of the coin landing tails up is

$$\frac{\text{the number of times the coin landed tails up}}{\text{the number of time the coin was flipped}}$$

$\therefore$ The fraction of success of the coin landing tails up $= \frac{9}{20}$

Let A be one possible outcome of a random experiment. If we conduct the experiment several times over under the same conditions, then

$$\text{the fraction of success of } A = \frac{\text{number of times the outcome } A \text{ occurs}}{\text{total number of times the experiment is conducted}}$$

• Obtaining the probability through observations

Consider a random experiment with several possible outcomes. Then the likelihood of each outcome is called the probability of that outcome.
The outcome resulting from flipping a fair coin once cannot be stated with certainty before the experiment is conducted. Let us consider the outcomes of this experiment when it is repeated a large number of times under the same conditions.

The outcomes that were obtained by performing the random experiment of flipping a Rs. 2 coin 20 times and observing the side that lands up have been recorded in the table given below and the table has been completed.

Number of times the experiment was conducted	Total number of times Heads occurred	Total number of times Tails occurred	Fraction of success of getting heads $= \frac{\text{number of heads}}{\text{number of times the coin was flipped}}$	Fraction of success of getting tails $= \frac{\text{number of tails}}{\text{number of times the coin was flipped}}$
1	1	0	$\frac{1}{1} = 1$	$\frac{0}{1} = 0$
2	1	1	$\frac{1}{2} = 0.5$	$\frac{1}{2} = 0.5$
3	1	2	$\frac{1}{3} = 0.33$	$\frac{2}{3} = 0.67$
4	2	2	$\frac{2}{4} = 0.5$	$\frac{2}{4} = 0.5$
5	2	3	$\frac{2}{5} = 0.4$	$\frac{3}{5} = 0.6$
6	2	4	$\frac{2}{6} = 0.33$	$\frac{4}{6} = 0.67$
7	3	4	$\frac{3}{7} = 0.43$	$\frac{4}{7} = 0.57$
8	4	4	$\frac{4}{8} = 0.5$	$\frac{4}{8} = 0.5$
9	4	5	$\frac{4}{9} = 0.44$	$\frac{5}{9} = 0.56$
10	5	5	$\frac{5}{10} = 0.5$	$\frac{5}{10} = 0.5$
11	5	6	$\frac{5}{11} = 0.45$	$\frac{6}{11} = 0.55$
12	5	7	$\frac{5}{12} = 0.42$	$\frac{7}{12} = 0.58$
13	5	8	$\frac{5}{13} = 0.38$	$\frac{8}{13} = 0.62$
14	6	8	$\frac{6}{14} = 0.43$	$\frac{8}{14} = 0.57$

Number of times the experiment was conducted	Total number of times Heads occurred	Total number of times Tails occurred	Fraction of success of getting heads $= \frac{\text{number of heads}}{\text{number of times the coin was flipped}}$	Fraction of success of getting tails $= \frac{\text{number of tails}}{\text{number of times the coin was flipped}}$
15	7	8	$\frac{7}{15} = 0.47$	$\frac{8}{15} = 0.53$
16	8	8	$\frac{8}{16} = 0.5$	$\frac{8}{16} = 0.5$
17	9	8	$\frac{9}{17} = 0.53$	$\frac{8}{17} = 0.47$
18	10	8	$\frac{10}{18} = 0.56$	$\frac{8}{18} = 0.44$
19	10	9	$\frac{10}{19} = 0.53$	$\frac{9}{19} = 0.47$
20	11	9	$\frac{11}{20} = 0.55$	$\frac{9}{20} = 0.45$

Activity 1

Complete the table by flipping a coin 40 times with your friends.

Number of times the coin was flipped	Number of times tails occurred	Number of times heads occurred	Number of times tails occurred / Number of times the coin was flipped	Number of times heads occurred / Number of times the coin was flipped

An important conclusion that we can arrive at through this experiment is that the fractions,

$\frac{\text{number of times tails occurred}}{\text{number of times the coin was flipped}}$ and $\frac{\text{number of times heads occurred}}{\text{number of times the coin was flipped}}$ approach the value $\frac{1}{2}$ as the number of times the coin is flipped is increased.

- Since the number of times an outcome of an experiment occurs is always less than or equal to the total number of times the experiment is repeated, the fraction of success of an outcome takes a value from 0 to 1. If the fraction of success of the outcome A approaches a constant value when the number of times the experiment is repeated (n) is increased, then that value is called the **experimental probability** of the outcome A occurring when the experiment is conducted once.

The sun always rises from the east, and does not depend on the number of times the sunrise is observed.
Therefore, the probability of the sun rising from the east is 1. The probability of the sun rising from the south is 0 because it never rises from the south.

- If an outcome will definitely occur, then irrespective of the value n (the number of times the experiment is conducted) takes, its fraction of success is $\frac{n}{n} = 1$. In this case, the probability of the outcome occurring is 1. Accordingly, the probability of an event that will definitely occur is 1.
- If an outcome cannot occur when an experiment is conducted, then, irrespective of the value n takes, its fraction of success is $\frac{0}{n} = 0$. Therefore, the probability of that outcome occurring is 0. Accordingly, the probability of an event that will definitely not occur is 0.

Apart from these two special cases, the probability of a possible outcome of a random experiment occurring is a value between 0 and 1.

★ When the probability of an outcome of a random experiment occurring is not known, then, the fraction of success that is obtained by increasing n, that is by conducting the experiment a large number of times, is value that is suitable to be used as an estimate of the probability of that outcome occurring.

Exercise 29.2

(1) There are three identical beads in a bag. They are red, blue and yellow in colour. A bead is drawn, its colour is recorded and is put back in the bag. Another bead is drawn, its colour is also recorded and is put back. The outcomes of this experiment which was repeated 50 times are given in the following table.

Bead	Number of times it was drawn
Red	18
Blue	17
Yellow	15

(i) Find the experimental probability of drawing the red bead.

(ii) Find the experimental probability of drawing the blue bead.

(iii) Find the experimental probability of drawing the yellow bead.

(2) An unbiased tetrahedral die with the numbers 1 to 4 marked on its four faces (one number on each face), was tossed 40 times and the number on the face that landed down was recorded. The results of this experiment are shown below.

Number	Number of times it occurred
1	8
2	11
3	10
4	11

(i) Find the experimental probability of getting the number 2.

(ii) Find the experimental probability of getting an even number.

(iii) Find the experimental probability of getting a prime number.

(iv) Find the experimental probability of getting a number greater than 1.

29.3 Theoretical Probability

Let us find the probability of the occurrence of each possible outcome of a random experiment with equally likely outcomes.

- In the random experiment of flipping an unbiased coin and observing the side that lands up, the outcomes are "the coin landing heads up" and "the coin landing tails up". The likelihood of each of these two outcomes occurring is the same.

- When an unbiased die is rolled, the number on the face that shows up is either 1 or 2 or 3 or 4 or 5 or 6. There is an equal likelihood of each of these outcomes occurring.

The probability of 2 showing up when an unbiased die is rolled can be calculated as follows.

The outcome can be either 1 or 2 or 3 or 4 or 5 or 6.
Since the die is unbiased, there is an equal likelihood of each of these six numbers showing up. Therefore, the probability of any one of the six numbers from 1 to 6 showing up is $\frac{1}{6}$.

Accordingly, the probability of 2 showing up = $\frac{1}{6}$

- Three of the six numbers on the die are even numbers. Therefore, the probability of an even number showing up is $\frac{3}{6} = \frac{1}{2}$.

When there is an equal likelihood of each of the possible outcomes of a random experiment occurring, the theoretical probability of a selected outcome occurring

$$= \frac{1}{\text{total number of possible outcomes of the random experiment}} .$$

The method of finding the theoretical probability of each outcome of a random experiment, when the probabilities of the possible outcomes of the experiment occurring are different to each other, is described in the following example.

Example 1

In an opaque bag, there are 4 red balls, 5 blue balls and 2 green balls which are identical in all aspects except the colour. Find the probability of a ball drawn from the bag

(i) being red.
(ii) being blue.
(iii) being green.

$$\text{Probability of being red} = \frac{\text{number of red balls}}{\text{total number of balls}} = \frac{4}{11}$$

$$\text{Probability of being blue} = \frac{\text{number of blue balls}}{\text{total number of balls}} = \frac{5}{11}$$

$$\text{Probability of being green} = \frac{\text{number of green balls}}{\text{total number of balls}} = \frac{2}{11}$$

Exercise 29.3

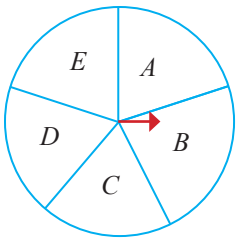
(1) An unbiased die with its faces marked from 1 to 6 is rolled. Find the probability of each of the following events.

(i) The number 5 showing up.
(ii) An even number showing up.
(iii) A square number showing up.

(2) There are 3 white beads 2 black beads and 1 blue bead in a bag, which are identical in size and shape. A bead is drawn randomly from the bag. Find the probability of each of the following events.

(i) Drawing a white bead
(ii) Drawing a black bead.
(iii) Drawing a blue bead.
(iv) Drawing a white bead or a black bead.
(v) Drawing a bead which is not black in colour.
(vi) Drawing a red bead.

(3) As shown in the figure, a circular lamina is divided into five equal parts and an indicator is fixed at the centre. The five parts are named *A, B, C, D* and *E*. When the indicator is rotated freely, it stops in one of the five parts. Accordingly, find the probability of each of the following events.



(i) The indicator stopping in *D*.
(ii) The indicator stopping in *A* or *D*
(iii) The indicator stopping in *B*, *C* or *E*.

Summary

- The likelihood of an event occurring is its probability.
- Let *A* be one possible outcome of a random experiment. If the experiment is repeated several times under the same conditions, then

$$\text{the fraction of success of } A = \frac{\text{number of times the outcome } A \text{ occurs}}{\text{total number of times the experiment is conducted}}.$$

- When there is an equal likelihood of each of the possible outcomes of a random experiment occurring, the theoretical probability of a selected outcome occurring is

$$\frac{1}{\text{total number of possible outcomes of the random experiment}}.$$