24 Construction of Rectilinear Plane Figures

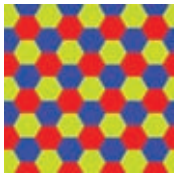
By studying this lesson you will be able to

- construct a straight line segment of given length,
- construct an equilateral triangle of given side length, and
- construct a hexagon by means of an equilateral triangle or a circle.

24.1 Constructions

The following figure presents examples of equilateral triangles and regular hexagons that can be observed around us.





Equilateral triangles and regular hexagons are two types of convex polygons that are important in geometry.

In geometry, it is necessary to draw as well as construct plane figures. When a plane figure is being drawn, it is done according to the given data, without paying much attention to the measurements. However, when a plane figure is being constructed, attention needs to be paid to the measurements, and a figure of the correct size should be constructed according to the given data.

Geometrical constructions are done with a pair of compasses and a straight edge.

When it is necessary to measure lengths and angles, then the appropriate measuring instruments need to be used.

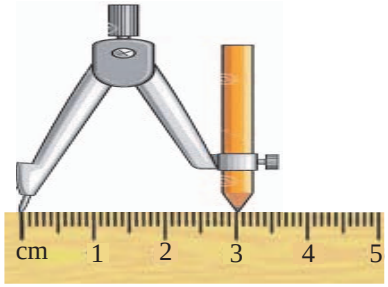
24.2 Construction of a straight line segment

You have learnt earlier that a straight line segment is a portion of a straight line.

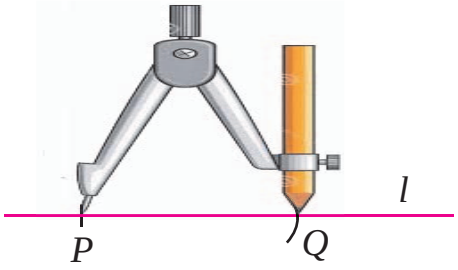
Now let us construct the straight line segment PQ of length 3 cm.

Step 1 - Draw a straight line using a ruler. Name it l. Mark a point on the straight line l and name it P.

Step 2 - Place the pair of compasses on the ruler and set it so that the point of the pair of compasses and the pencil point are at a distance of 3 cm apart.



Step 3 - Place the point of the pair of compasses on the point P and mark a point on the line which is 3 cm from P and name it Q.



Step 4 - Write "3 cm" between the two points P and Q.

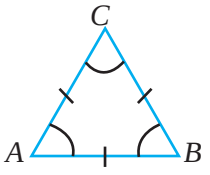
Now you have constructed a straight line segment PQ of length 3 cm. To indicate that the length of this straight line segment is 3 cm, we write $PQ = 3$ cm.

➤ Construct straight line segments of the lengths given below.

(i) $AB = 7$ cm (ii) $XY = 7.8$ cm

24.3 Construction of an equilateral triangle

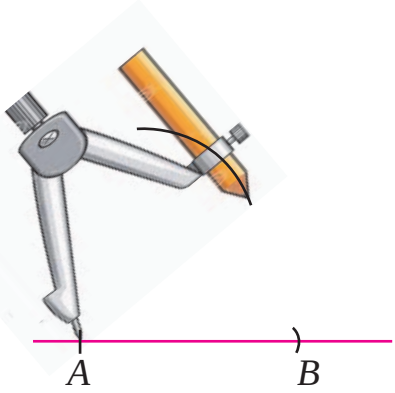
You have learnt earlier that an equilateral triangle is a triangle with sides which are equal in length and angles which are equal in magnitude.



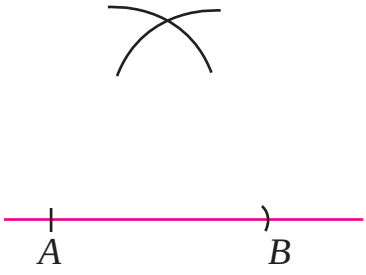
Let us construct an equilateral triangle of side length 3 cm.

Step 1 - Construct a straight line segment *AB* of length 3 cm using a pair of compasses and a ruler.

Step 2 - Set the pair of compasses so that the point of the pair of compasses and the pencil point are at a distance of 3 cm apart. Place the point of the pair of compasses on the point *A* and construct an arc as shown in the figure.

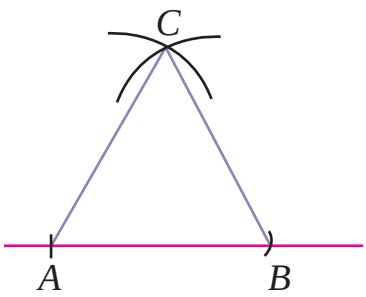


Step 3 - Place the point of the pair of compasses at the point *B* and construct another arc such that it intersects the first arc. If the arcs do not intersect, place the point of the pair of compasses at *A* and lengthen the initial arc.



Name the point of intersection of the two arcs as *C*.

Step 4 - Join *AC* and *BC*.



Then you will obtain the equilateral triangle *ABC* of side length 3 cm.

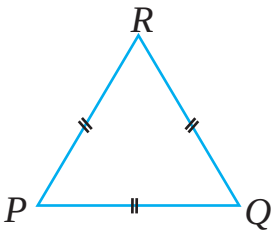
➤ (i) Construct two equilateral triangles of side length 4 cm and 5.7 cm.
(ii) Measure the angles of the above two triangles that you constructed.

Exercise 24.1

(1) Construct the straight line segment *LM* of length 6 cm using a straight edge and a pair of compasses.

(2) Draw the straight line *l* and construct the straight line segment *PQ* of length 7.5 cm on it.

(3) (i) Construct the equilateral triangle *PQR* in the figure. Measure and write down the magnitude of the angle *PQR*.



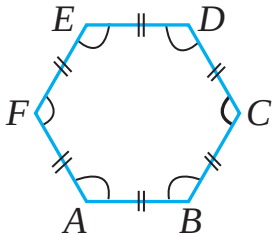
(ii) Mark the mid points of the sides of the triangle *PQR* and name them *X*, *Y* and *Z*. Draw the triangle *XYZ*.

(4) (i) Cut out 6 equilateral triangles, each of side length 3 cm, from different coloured paper.

(ii) Mark a point *O* on a piece of paper and paste the triangles on this paper such that one vertex of each triangle coincides with *O* and adjacent triangles have a side which touches one side of each triangle next to it. What is the shape of the figure you obtain by doing this?

24.4 Constructing a regular hexagon

In the figure is a regular hexagon *ABCDEF*. A hexagon is a closed convex polygon bounded by 6 straight line segments. In a regular hexagon,



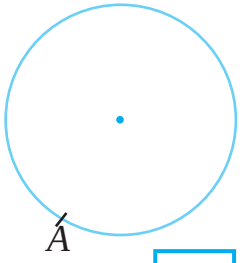
- the sides are of equal length
- the angles are of equal magnitude

Now let us see how a regular hexagon is constructed.

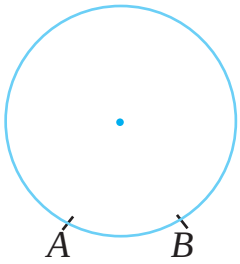
• Constructing a regular hexagon by means of a circle

Step 1 - Construct a circle of radius 1.5 cm using a pair of compasses.

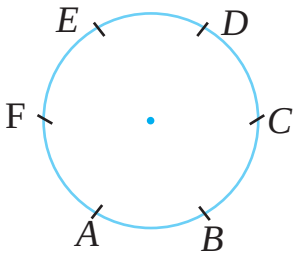
Step 2 - Mark a point *A* on this circle.



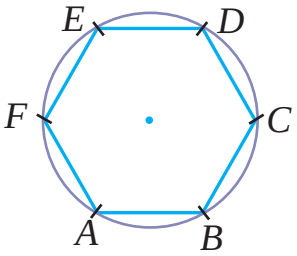
Step 3 - Set the pair of compasses so that the point of the pair of compasses and the pencil point are at a distance of 1.5 cm apart. Place the point of the pair of compasses on point *A*, draw an arc which intersects the circle and name this point *B*.



Step 4 - Similarly, place the point of the pair of compasses on the point *B* and mark the point *C*. Now, place the point on *C* and mark the point *D*, place the point on *D* and mark the point *E* and finally place the point on *E* and mark the point *F*.



Step 5 - Join the points *A*, *B*, *C*, *D*, *E* and *F* respectively.



You have now constructed the regular hexagon *ABCDEF* of side length 1.5 cm. By measuring the angles of the regular hexagon establish the fact that they are of equal magnitude.

➤ Construct a regular hexagon of side length 3.5 cm by following the above steps.

• Constructing a regular hexagon using an equilateral triangle

Step 1 - Construct the equilateral triangle *ABC* of side length 4 cm.
Step 2 - Construct the equilateral triangle *BCD* by taking *BC* as a side.
Step 3 - Construct the equilateral triangle *CDE* by taking *CD* as a side.
Step 4 - Construct the equilateral triangle *CEF* by taking *CE* as a side.
Step 5 - Construct the equilateral triangle *CFG* by taking *CF* as a side.
Step 6 - Join *A* and *G*.

Then you will obtain the regular hexagon *ABDEFG* of side length 4 cm. A regular hexagon of any side length can be constructed in the above manner.

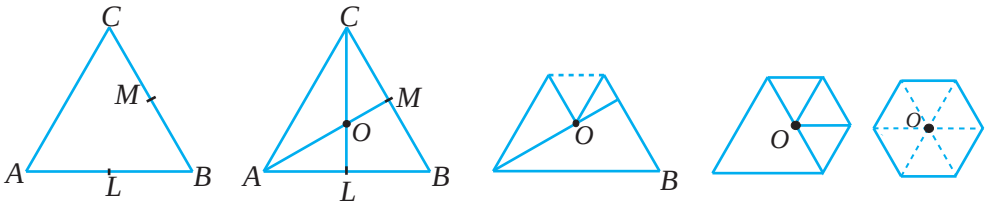
➤ Construct a regular hexagon of side length 3 cm.

Activity 1

Step 1 - Construct the equilateral triangle *ABC* of side length 3 cm on a piece of paper.

Step 2 - Mark the midpoint of *AB* as *L* and the midpoint of *BC* as *M*.

Step 3 - Join *MA* and *LC*. Name the point of intersection of *LC* and *MA* as *O*. Cut out the triangular lamina *ABC*.



Step 4 - Fold the triangle such that each vertex coincides with *O*. The figure that is obtained by doing this is a regular hexagon.

Step 5 - Measure the length of a side of the regular hexagon.

- The length of a side of the regular hexagon is 1 cm.
- That is, the length of a side of the original equilateral triangle is 3 times the length of a side of the regular hexagon.

➤ Construct a regular hexagon of side length 3 cm by following the above steps.

Exercise 24.2

(1) (i) Construct the circle of radius 5 cm and centre *O*.

(ii) Construct the regular hexagon *ABCDEF* of side length 5 cm with its vertices on the above circle.

(iii) Join *OA*, *OB*, *OC*, *OD*, *OE* and *OF*. How many triangles do you get? Are they all equilateral triangles?

(2) Construct a regular hexagon of side length 6 cm.

(3) (i) Construct a straight line segment AB of length 5 cm.
(ii) Create two equilateral triangles, each of which has AB as a side.

(4) (i) Construct a circle of radius 4 cm.
(ii) Construct a regular hexagon with its vertices on the above circle.
(iii) By producing three sides of the hexagon obtain an equilateral triangle.

(5) (i) Construct a circle of radius 5 cm.
(ii) Construct a regular hexagon with its vertices on the above circle.
(iii) Construct three equilateral triangles on three sides of the hexagon, leaving out a side of the hexagon between two triangles.
(iv) What is the shape of the total figure?

Summary

- The construction of an equilateral triangle can be done in four steps.
  - Construct a straight line segment.
  - Taking the same length as the straight line segment onto the pair of compasses, construct an arc placing the point of the pair of compasses at one end of the line segment.
  - Construct an arc from the other end point, using the same length as above, such that it intersects the earlier arc.
  - Join the intersection point to the end points of the straight line segment.
- A regular hexagon can be constructed by performing the following steps.
  - Construct a circle.
  - Divide the circle into 6 equal parts by intersecting the circle with arcs of the same length as the radius of the circle.
  - Join the points of intersection.