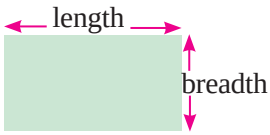
19

Volume

By studying this lesson you will be able to

- identify what volume means,
- identify the different units used to measure volume, and
- find the volume of a cube and a cuboid.

19.1 Identifying what volume means



You have learnt that area is the extent of a plane surface.
Now let us see what volume is.

Let us consider the following objects.

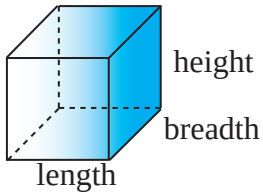




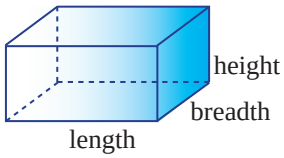

Each of the above objects occupies a certain amount of space. This space is called the **volume** of the object.

Now let us consider a cube and a cuboid.

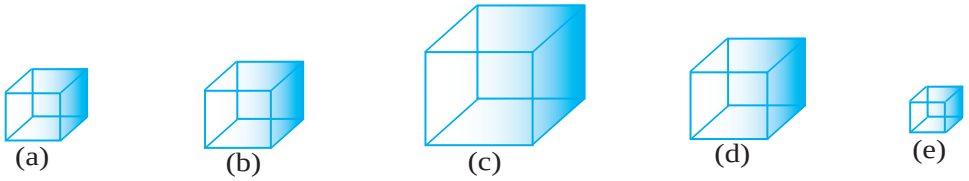
A cube consists of 6 equal square shaped faces. It has 12 edges of equal length. As shown in the figure, the length, breadth and height of a cube are equal.



A cuboid consists of three pairs of rectangular plane surfaces; each pair being equal. It has three sets of 4 edges of equal length; totalling 12 edges. As shown in the figure, the length, breadth and height can be different to each other.



The following figure depicts five cubes.



When these cubes are arranged in ascending order of their volumes we obtain *e*, *a*, *b*, *d*, *c*.

Activity 1

Step 1 - Collect at least 4 solid cube or cuboid shaped objects.

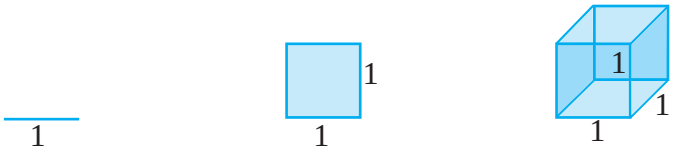
Step 2 - See whether you can arrange them in increasing order of their volumes.

Step 3 - Inquire from your teacher whether the order in which you arranged the objects is correct.

19.2 Measuring the volume of solid objects using arbitrary units

By comparing the amount of space occupied by a die with the amount of space occupied by a brick, we can easily say that the volume of the brick is greater than the volume of the die.

However, it is difficult to compare the volumes of objects such as statues and logs which are of different shapes by just observing them. Therefore let us consider the units that are used to measure volumes.



A straight line segment of length 1 unit

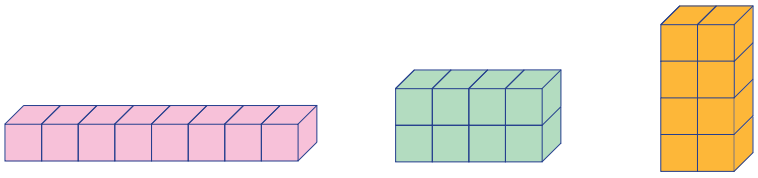
A square of area 1 square unit

A cube of volume 1 cubic unit

The unit to measure area is 1 square unit which is the area of a square of side length 1 unit.

The unit to measure volume is 1 cubic unit which is the volume of a cube of side length 1 unit.

The following figure depicts a few cuboids that have been created using 8 identical cubes. Now let us find the volume of each of these cuboids.



(a) (b) (c)

Let us take the volume of a small cube to be 1 cubic unit. Then,

since there are 8 cubes in figure (a), the volume of the cuboid in figure (a) is 8 cubic units,

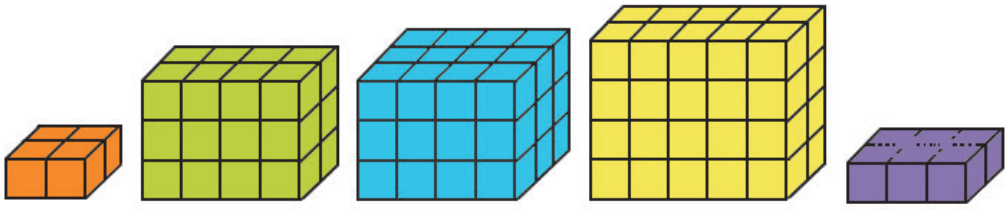
since there are 8 cubes in figure (b), the volume of the cuboid in figure (b) is 8 cubic units, and

since there are 8 cubes in figure (c), the volume of the cuboid in figure (c) is 8 cubic units.

Although the length, breadth and height of these cuboids take different values, their volumes are all equal.

Exercise 19.1

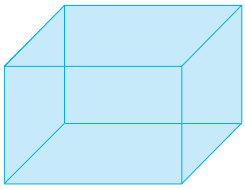
(1) Find the volume of each of the solid objects in the given figure by counting the number of small cubes each object contains. Consider the volume of a small cube to be 1 cubic unit.

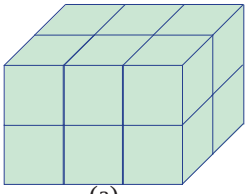


(a) (b) (c) (d) (e)

• More on measuring the volume of solid objects using arbitrary units

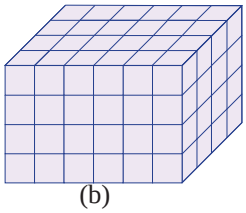
Consider how the volume of the cuboid shown below has been found.





(a)

Here the cuboid has been divided into 12 smaller cubes of side length 1 unit. Let us take the volume of one small cube to be 1 cubic unit. Then the volume of the cuboid is 12 cubic units.



(b)

Here the cuboid has been divided into 96 small cubes of side length 1 unit. Let us take the volume of one small cube to be 1 cubic unit. Then the volume of this cuboid is 96 cubic units.

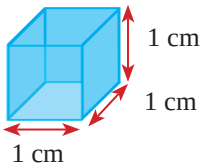
Understand that the volume of the small cube that we used as our unit to measure volume is different in the above two cases. Accordingly, two different numerical values were obtained for the volume of the cuboid.

As indicated above, an arbitrary unit can be used to measure the volume of a solid object. It is important to mention the unit that was used when writing the volume of an object, as the numerical value depends on the unit used, as seen above.

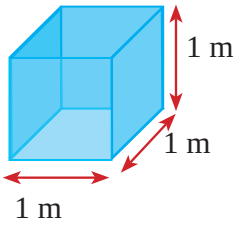
19.3 Standard units used to measure volume

We obtained different numerical values for the volume of a solid object, which depended on the unit that was used. To avoid this variance, standard units are used to measure volumes.

The volume of a cube of side length 1 cm is used as the standard unit of volume. It is defined as 1 cubic centimetre and written as 1 cm^3.

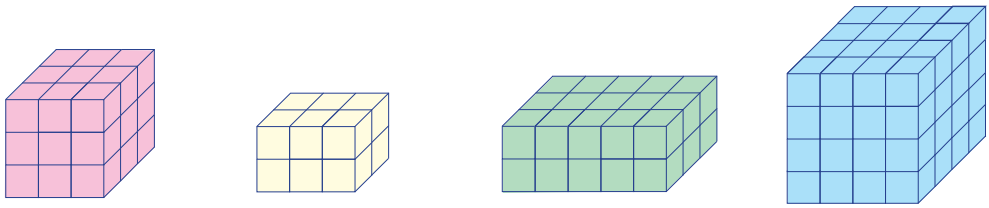


The volume of a cube of side length 1 metre is used as the unit to measure larger volumes. Its volume is 1 cubic metre. One cubic metre is written as 1 m^3.



Exercise 19.2

(1) Find the volume of each of the following solid objects in cubic centimetres. Consider the volume of a small cube to be 1 cm^3.



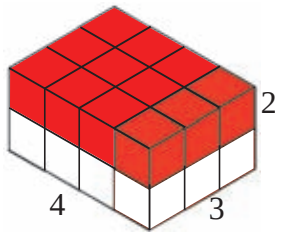
(a) (b) (c) (d)

19.4 Another method of finding the volume of a cube or a cuboid

Let us consider an easier method of finding the volume of a cube and a cuboid.

• The volume of a cuboid

A cuboid of length 4 units, breadth 3 units and height 2 units is shown here.



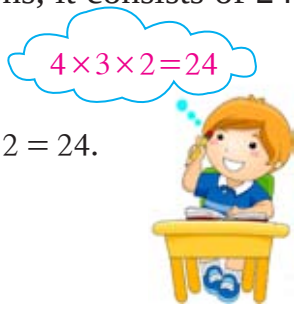
The portion highlighted in red consists of 12 cubes of volume 1 cubic unit each.

$4 \times 3 = 12$

Since the whole cuboid consists of two such portions, it consists of 24 cubes of volume 1 cubic unit each.

$12 \times 2 = 24$

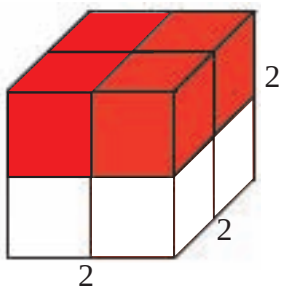
Therefore, the volume of the whole cuboid $= 4 \times 3 \times 2 = 24$.



Volume of a cuboid = length × breadth × height

• The volume of a cube

A cube of side length 2 units is shown here.



The portion highlighted in red consists of 4 cubes of volume 1 cubic unit each.

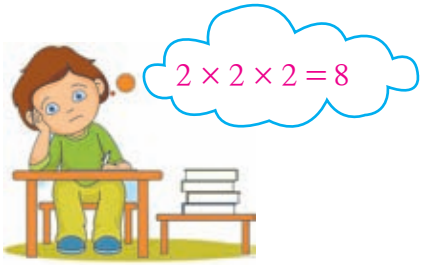
$2 \times 2 = 4$

Since the whole cube consists of two such portions, it consists of 8 cubes of volume 1 cubic unit each.

$4 \times 2 = 8$

Therefore,

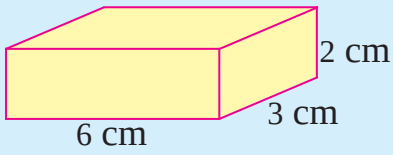
the volume of the whole cube of side length 2 units $= 2 \times 2 \times 2 = 8$



Volume of the cube = length × breadth × height
= side length × side length × side length
= (side length)^3

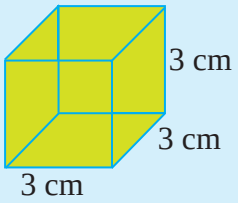
Example 1

Find the volume of the cuboid in the figure.



Length of the cuboid = 6 cm
Breadth of the cuboid = 3 cm
Height of the cuboid = 2 cm
Volume of the cuboid = length × breadth × height
= 6 cm × 3 cm × 2 cm
= 36 cm^3

Example 2



Find the volume of the cube in the figure.

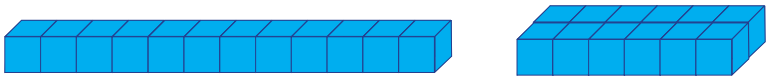
Volume of the cube = length × breadth × height

= 3 cm × 3 cm × 3 cm

= 27 cm^3

Exercise 19.3

(1) The following figure depicts two cuboids that have been formed using 12 cubes of volume 1 cm^3 each.



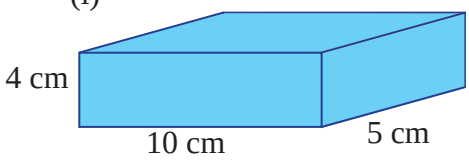
(i) Find the volume of each cuboid.

(ii) Find the length, breadth and height of each cuboid.

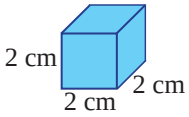
(iii) Write the length, breadth and height of another cuboid of volume 12 cm^3.

(2) Calculate the volume of each of the following solids.

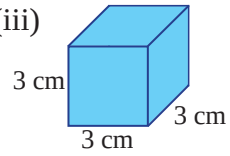
(i)



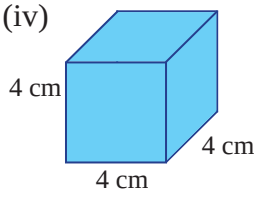
(ii)



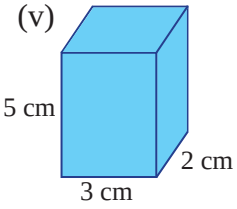
(iii)



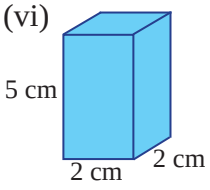
(iv)



(v)

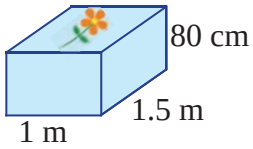


(vi)



(3) The volume of a cuboid shaped box is 60 cm^3. The length and breadth of the box are respectively 6 cm and 2 cm. Calculate its height.

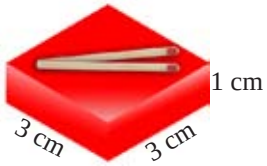
(4) The length, breadth and height of a cuboid shaped container are 1.5 m, 1 m and 80 cm respectively.



(i) Find the height of the container in centimetres.

(ii) Find the volume of the container in cubic centimetres.

(5) The figure shows a matchbox of length, breadth and height equal to 3 cm, 3 cm and 1 cm respectively.



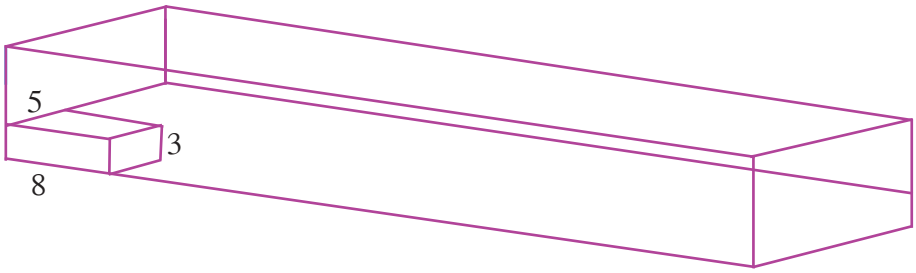
(i) Find the volume of this matchbox.

(ii) There are three layers, each consisting of 4 matchboxes in a package containing 12 of these matchboxes. Find the length, breadth and height of this package.

(iii) Show that the volume of this package is 108 cm^3.

19.5 Estimation of Volume

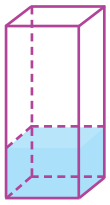
The length, breadth and height of a cake of soap are 8 cm, 5 cm and 3 cm respectively. The maximum number of cakes of soap that can be packed in the given box is 92. Estimate the volume of the box.



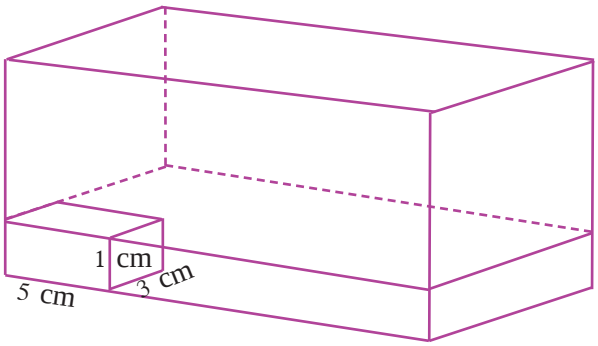
The volume of a cake of soap is approximately $8 \times 5 \times 3$ cm^3; that is, 120 cm^3. Therefore, the volume of the box is approximately 120×92 cm^3, that is, 11 040 cm^3.

Exercise 19.4

(1) The volume of the shaded cuboid portion in the figure is 16 cm^3. Estimate the volume of the whole cuboid.



(2) The length, breadth and height of a matchbox are 5 cm, 3 cm and 1 cm respectively. Matchboxes are packed in the box as shown in the figure. Estimate the volume of the box.



Summary

- The volume of a solid is the amount of space it occupies.
- Arbitrary units can be used to measure volumes. When stating the volume, the units used should also be mentioned.
- A cube of side length 1 cm is used as the standard unit of volume.
- Cubic centimetre (cm^3) and cubic metre (m^3) are two units that are used to measure volumes.
- The volume of a cuboid of length, breadth and height equal to a, b and c units respectively is $a \times b \times c$ cubic units.
- The volume of a cube of side length a units = a^3 cubic units.