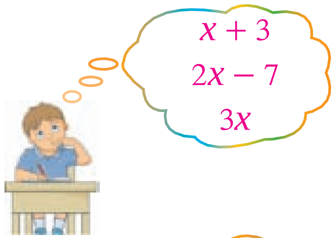
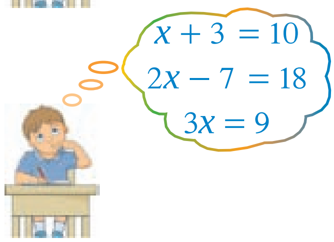
15 Equations and Formulae

By studying this lesson you will be able to
- construct a simple equation in one unknown,
- solve simple equations,
- construct simple formulae, and
- find the value of any variable of a formula by substituting positive whole numbers for the other variables.

15.1 Constructing simple equations



You have learnt earlier to construct algebraic expressions by using algebraic symbols for unknown values, numbers for known values and operations.



When the value represented by an algebraic expression is equal to the value of a given number, it can be expressed as "algebraic expression = number".

When the value represented by an algebraic expression is equal to the value represented by another algebraic expression, it can be expressed as
"first algebraic expression = second algebraic expression".

Relationships of the above forms are called **equations**.

Consider the equations $x + 3 = 10$, $2x - 7 = 18$ and $3x = 9$. There is only one unknown in each of these equations. The index of the unknown in each equation is also 1.

An equation containing only one unknown of index one is called a **simple equation**.

In the equation $x + 5 = 8$, the value of the algebraic expression $x + 5$ on the left-hand side has been equated to 8 on the right-hand side.

An equation always contains the symbol "=". Apart from this, it also contains unknown terms, numbers and operations.

- A vendor had x mangoes. He bought another 24 mangoes. He now has a total of 114 mangoes. Let us express this information by an equation.

The number of mangoes the vendor had initially $= x$
The number of mangoes the vendor bought $= 24$
The total number of mangoes the vendor has now $= x + 24$
Moreover, since the total number of mangoes the vendor has now is 114,

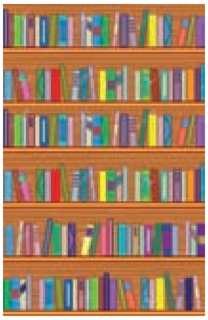
$$x + 24 = 114$$

- "The price of a loaf of bread reduced by 4 rupees. The new price of a loaf of bread is 50 rupees." Let us express this by an equation.

Let us take the initial price of a loaf of bread as b rupees.
Since the price of a loaf of bread reduced by 4 rupees,
the new price of a loaf of bread $= b - 4$
Moreover, since the new price of a loaf of bread is 50 rupees,

$$b - 4 = 50$$

- Books have been placed on the shelves of a bookcase in a library such that each shelf contains x books. There are 6 shelves in the bookcase. After 10 of these books were issued to students, the number of books remaining in the bookcase reduced to 104. Let us express this information by an equation.



The total number of books that were on the 6 shelves $= 6x$
Number of books issued to students $= 10$
$\therefore$ the number of books remaining in the bookcase $= 6x - 10$
Moreover, since the number of books remaining is 104,

$$6x - 10 = 104$$

Example 1

When 13 is added to twice the value of a given number, the value that is obtained is 85. Represent this information by an equation.

Let us take the number as a.

Twice this number $= 2 \times a = 2a$

The number that is added $= 13$

The number that is obtained $= 2a + 13$

Moreover, since the number that is obtained is 85,

$$2a + 13 = 85$$

Example 2

In a certain year, a father was three times the age of his daughter on the day of her wedding. The girl's mother is 4 years younger to her father. That year, the mother's age was 62. By taking the age of the daughter on the day of her wedding as x years, construct an equation to represent this information.

Three times the age of the girl on the day of her wedding $= 3x$

$\therefore$ the father's age that year $= 3x$

The age of the mother who is four years younger to the father $= 3x - 4$

Moreover, since the mother's age that year was 62,

$$3x - 4 = 62$$

Exercise 15.1

(1) Each part, construct an equation to represent the information given in the statements.

(i) When 7 is added to the number represented by x, the value obtained is 20.

(ii) Nimal's present age is x years. In another 5 years he will be 18 years old.

(iii) When 12 is subtracted from the number represented by y, the value obtained is 27.

(iv) Saman received a salary of x rupees in January. The amount remaining from his salary after he had sent Rs 5000 to his mother was Rs 8000.

(v) When a number x is multiplied by 2, the value that is obtained is 34.

(vi) The amount spent on three pencils of the same type which cost p rupees each was 54 rupees.

(vii) The price of 1 kg of rice is r rupees. When 80 rupees is added to the price of 4 kg of rice, the value is 500 rupees.

(viii) The age of a father was three times the age of his son on the day of the son's wedding. The mother's age on this day was 60 years. The mother is 6 years younger to the father. Take the son's age to be x years.

(ix) Due to the price of a newspaper increasing by 10 rupees, its price is now 30 rupees.

(x) When a piece of cloth of length 70 cm is cut out, a piece of length 40 cm is left over.

(xi) 200 rupees was needed to purchase 5 mangosteens and one pineapple priced at 100 rupees.

(xii) When 12 is subtracted from five times a certain number, the value obtained is 98.

(xiii) When 4 is added to three times a certain number, the value obtained is 73.

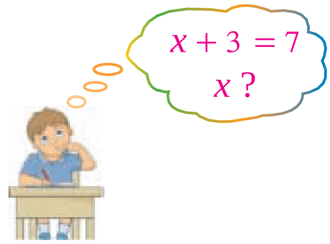
(xiv) Sameera needed to buy a book worth 500 rupees. He saved an equal amount of money each day for 7 days. To purchase the book, he needed to add another 129 rupees to the amount he had saved.

15.2 Solving simple equations

The equality symbol "=" in an equation expresses the fact that the value represented on the left-hand side of the symbol is equal to the value represented on the right-hand side.

What we mean by solving a simple equation is finding the value of the unknown term which satisfies the equation (that is, the value for which the equation holds true). This value is called the solution of the equation. A simple equation has only one solution.

For example, when 4 is substituted for x in the equation $x + 3 = 7$, the left-hand side of the equation is equal to the right-hand side.



Therefore, the solution to the equation $x + 3 = 7$ is $x = 4$.

• Solving simple equations using algebraic methods

You have learnt that the equality symbol "=" in an equation expresses the fact that the value on the left-hand side of this symbol is equal to the value on the right-hand side.

When solving simple equations, the value that the unknown should take for the left-hand side to be equal to the right-hand side of the equation can be found as follows.

➢ Let us find the value of the unknown which satisfies the equation $a + 8 = 10$.

When the same number is subtracted from the two sides of an equation, the new values that are obtained on the two sides are equal.

Let us subtract 8 from both sides of the equation $a + 8 = 10$.

$$a + 8 - 8 = 10 - 8 \qquad (8 - 8 = 0)$$
$$\therefore\ a = 2$$

➢ Let us find the value of the unknown which satisfies the equation $x - 7 = 10$.

In this equation, the value of $x - 7$ is equal to 10.

When the same number is added to the two sides of an equation, the new values that are obtained on the two sides are equal.

When 7 is added to the two sides of the equation $x - 7 = 10$, the left-hand side is equal to x and the right-hand side is equal to 17.

$$x - 7 + 7 = 10 + 7 \qquad (-7 + 7 = 0)$$
$$\therefore\ x = 17$$

- Let us solve the equation $5x = 10$.

When the two sides of an equation are divided by the same non-zero number, the new values that are obtained on the two sides are equal.

Let us divide both sides of the equation $5x = 10$ by 5.

$$\frac{5x}{5} = \frac{10}{5} \qquad (\frac{5}{5} = 1)$$
$$\therefore \quad x = 2$$

When the value that is obtained is substituted for the unknown in the equation and simplified, if the same number is obtained on the two sides of the equation, then the accuracy of your answer is established.

Let us establish this, through the following examples.

Example 1

Solve $3y - 2 = 10$.

$$3y - 2 = 10$$
$$3y - 2 + 2 = 10 + 2 \quad \text{(let us add 2 to both sides)} \; (-2 + 2 = 0)$$
$$3y = 12$$
$$\frac{3y}{3} = \frac{12}{3} \quad \text{(let us divide both sides by 3)} \; (\frac{3}{3} = 1)$$
$$\therefore \; y = 4$$

Let us examine whether the solution $y = 4$ that you obtained is correct.
When $y = 4$,

Left-hand side $= 3y - 2$
$= 3 \times 4 - 2$
$= 12 - 2$
$= 10$

Right-hand side $= 10$

Therefore, left-hand side = right-hand side

Therefore, the solution $y = 4$ is correct.

Example 2

It costs 96 rupees to buy four books of the same price and 3 pencils priced at 8 rupees each. Find the price of a book.

Let us take the price of a book as x rupees.

Then the price of four books = $4x$ rupees

The price of 3 pencils, each priced at 8 rupees = 3×8 rupees = 24 rupees

Therefore, $4x + 24 = 96$

$$4x + 24 - 24 = 96 - 24$$

$$4x = 72$$

$$\frac{4x}{4} = \frac{72}{4}$$

$$x = 18$$

$\therefore$ the price of a book is 18 rupees.

Let us examine whether the solution $x = 18$ is correct.

When $x = 18$,

Left-hand side $= 4x + 24$

$= 4 \times 18 + 24 = 72 + 24 = 96$

Right-hand side $= 96$

That is, left-hand side = right-hand side

$\therefore$ the solution $x = 18$ is correct.

• Another method of solving simple equations

The inverse operations of the mathematical operations addition, subtraction, multiplication and division which we use in equations are respectively subtraction, addition, division and multiplication.

Another method of solving a simple equation of the above form is performing the inverse operations of the operations on the left-hand side, on the value on the right-hand side.

Let us solve the equation $3x + 7 = 10$.

The left-hand side of this equation is $3x + 7$.

The right-hand side is 10.

$x \xrightarrow{} \boxed{\times 3} \xrightarrow{} 3x \xrightarrow{} \boxed{+7} \xrightarrow{} 3x + 7$ (left-hand side)

$x \xleftarrow{} \boxed{\div 3} \xleftarrow{} 3x \xleftarrow{} \boxed{-7} \xleftarrow{} 3x + 7$

$1 \xleftarrow{} \boxed{\div 3} \xleftarrow{} 3 \xleftarrow{} \boxed{-7} \xleftarrow{} 10$ (right-hand side)

$\therefore x = 1$

Example 1

Solve $x - 7 = 10$.

$x \xrightarrow{} \boxed{-7} \xrightarrow{} x - 7$ (left-hand side)

$\underset{17}{x} \xleftarrow{} \boxed{+7} \xleftarrow{} \underset{10}{x - 7}$ (right-hand side)

$\therefore x = 17$

Example 2

Solve $5x = 30$.

$x \xrightarrow{} \boxed{\times 5} \xrightarrow{} 5x$ (left-hand side)

$\underset{6}{x} \xleftarrow{} \boxed{\div 5} \xleftarrow{} \underset{30}{5x}$ (right-hand side)

$\therefore x = 6$

Example 3

Solve $3y - 2 = 10$.

$y \xrightarrow{} \boxed{\times 3} \xrightarrow{} 3y \xrightarrow{} \boxed{-2} \xrightarrow{} 3y - 2$ (left - hand side)

$\underset{4}{y} \xleftarrow{} \boxed{\div 3} \xleftarrow{} \underset{12}{3y} \xleftarrow{} \boxed{+2} \xleftarrow{} \underset{10}{3y - 2}$ (right-hand side)

$\therefore y = 4$

Exercise 15.2

(1) Solve each of the following equations.

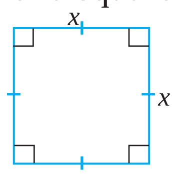
(i) $x + 6 = 7$ (ii) $x + 4 = 20$ (iii) $x - 5 = 14$ (iv) $x - 3 = 27$
(v) $6x = 48$ (vi) $7b = 56$ (vii) $2x + 5 = 9$ (viii) $8x + 7 = 79$
(ix) $7x - 5 = 51$ (x) $9x - 7 = 101$ (xi) $11x + 1 = 12$

(2) The price of a banana in a comb with 18 fruits is y rupees. If it costs 170 rupees to buy this comb of bananas and a pineapple priced at 80 rupees, find the value of y.

15.3 Formulae

Let us develop the relationship between the length of a side of a square and the perimeter of the square.

Let the length of a side of a square be x cm and the perimeter of the square be p cm.



Since the perimeter of a square is the sum of the lengths of the four sides of the square,

$p = x + x + x + x \quad = 4x$

Equations such as the above are called **formulae**.

Here p is called the subject of the formula.

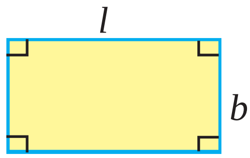
Accordingly, the relationship between the perimeter and the length of a square of length x units and perimeter p units is $p = 4x$.

This formula can be used to find the perimeter of any square of which the side length is known.

Since the units of the values on the two sides of a formula are the same, it is not necessary to state the units.

Formula for the perimeter of a rectangular lamina can also be developed as above.

Let the length of this rectangular lamina be l units, the breadth be b units and the perimeter be P in the same units.



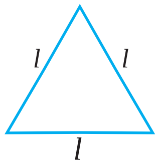
Then $P = l + b + l + b$

This can be written either as $P = 2l + 2b$ or $P = 2(l + b)$.

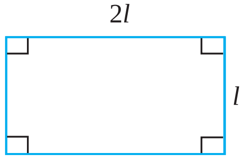
This formula can be used to find the perimeter of any rectangle of which the length and breadth are known.

Exercise 15.3

(1) The perimeter of an equilateral triangle of side length l units is P units. Construct a formula expressing the relationship between P and l.



(2) The breadth and the length of the given rectangle are l units and $2l$ units respectively. Construct a formula for its perimeter P in terms of l.



(3) The breadth of the rectangle in the figure is x cm. If the length is 10 cm more than the breadth and the perimeter is P in the same units, construct a formula expressing the relationship between P and x.

(4) There is a fixed charge of 100 rupees in the monthly electricity bill of a certain city. Apart from this, households that consume less than 100 units of electricity per month have to pay an additional amount of 8 rupees per unit consumed. If the monthly electricity bill of a consumer who uses n units (where $n < 100$) during a month is p rupees, construct a formula for p in terms of n.

(5) A machine produces N milk packets during the first hour. Every hour thereafter, it produces n packets. If T packets are produced in t hours, construct a formula for T in terms of n, N and t.

15.4 Substituting numerical values for the variables in a formula

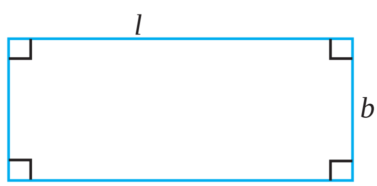
If the length, breadth and perimeter of a rectangle are l, b and P respectively, then $P = 2l + 2b$.

The length and the breadth of a certain rectangle are 13 cm and 7 cm respectively. Let us calculate its perimeter by using the above formula.

$$P = 2l + 2b$$

$l = 13$ cm and $b = 7$ cm

$$\begin{aligned}\text{Therefore, } P &= 2 \times 13 + 2 \times 7 \text{ cm} \\ &= 26 + 14 \text{ cm} \\ &= 40 \text{ cm}\end{aligned}$$



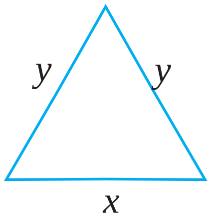
Exercise 15.4

(1) Find the value of N when $Q = 13$ and $D = 20$ in the formula $N = 18 + QD$.

(2) If the area of a square lamina of side length x units is A square units, a formula for A, in terms of x is $A = x^2$. Find the value of A when $x = 8$.

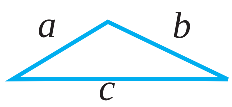
(3) (i) If the perimeter of the given triangle is P, develop a formula for P.

(ii) Find the value of P when $x = 16$ cm and $y = 12$ cm.



(4) (i) If the perimeter of the given triangle is P, construct a formula for P.

(ii) Find the value of P when $a = 4$ cm, $b = 5$ cm and $c = 6$ cm.



(5) If the area of a rectangular lamina of length l units and breadth b units is A square units, the formula for A, in terms of l and b is $A = lb$. Find the value of A when $l = 6$ cm and $b = 3$ cm.

Summary

- The relationship that is obtained when two mathematical expressions, of which at least one is algebraic, are equated to each other is an equation.
- The solution of an equation is the value of the unknown for which the equation holds true.
- A relationship between several variables can be expressed as a formula.
- The value of the any variable of a formula can be found by substituting positive whole numbers for the other variables.