10 Fractions

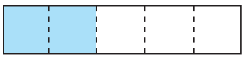
(Part I)

By studying this lesson you will be able to

- identify mixed numbers and improper fractions, and
- convert a mixed number into an improper fraction and an improper fraction into a mixed number.

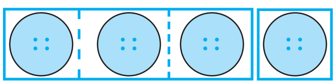
10.1 Fractions

Let us take the area bounded by the figure given below as one unit.



This unit has been divided into five equal parts, of which two have been coloured. As we have learnt previously, the amount coloured is $\frac{2}{5}$.

Similarly, if we take the four buttons shown below as one unit, we known that an amount of 3 buttons is $\frac{3}{4}$ of the amount of buttons there are.



Of the 25 children in a class, 13 are girls. When the number of girls in the class is written as a fraction of the total number of children in the class we obtain $\frac{13}{25}$. Here the total number of children in the class, that is 25, has been taken as one unit.

When a fraction is written numerically in this manner, the number below the line is called the **denominator** and the number above the line is called the **numerator**.

$$\frac{3}{4} \begin{array}{l} \leftarrow \text{Numerator} \\ \leftarrow \text{Denominator} \end{array}$$

Numbers such as $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{2}{5}$ which are smaller than one but larger than zero are called **proper fractions**. In a proper fraction, the numerator is always smaller than the denominator.

Proper fractions such as $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, with the numerator equal to 1, are defined as **unit fractions**.

Any fraction can be written in terms of its corresponding unit fraction.

For example,

$\frac{2}{3}$ is two $\frac{1}{3}$ s.

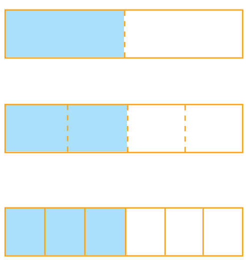
$\frac{5}{17}$ is five $\frac{1}{17}$ s.

Next let us recall what we have learnt about equivalent fractions.

$\frac{1}{2}$

$\frac{2}{4}$

$\frac{3}{6}$



Let us consider these three figures. The amounts that have been coloured in these figures are equal. That is, the fractions $\frac{1}{2}$, $\frac{2}{4}$ and $\frac{3}{6}$ that are represented by these figures are equal to each other.

That is, $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$.

We learnt in grade 6 that such fractions, which have denominators which are different to each other and numerators too which are different to each other, but which represent the same number are defined as equivalent fractions.

A fraction equivalent to a given fraction can be obtained by multiplying both the numerator and the denominator of the given fraction by a whole number (other than 0). Two such examples are given below.

$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

A fraction equivalent to a given fraction can also be obtained by dividing both the numerator and the denominator of the given fraction by a whole number (other than 0) which divides the numerator and the denominator without remainder.

Let us find a fraction equivalent to $\frac{18}{24}$. Let us divide the numerator and the denominator of $\frac{18}{24}$ by 3 which divides 18 and 24 without remainder.

$$\frac{18}{24} = \frac{18 \div 3}{24 \div 3} = \frac{6}{8}$$

Do the following review exercise to recall the facts you have learnt about fractions.

Review Exercise

(1) Select the unit fractions from the following proper fractions and write them down.

$\frac{2}{3}, \frac{1}{7}, \frac{4}{15}, \frac{1}{3}, \frac{1}{100}$

(2) Fill in the blanks by selecting the suitable value from within the brackets.

(i) $\frac{3}{5}$ is ...... $\frac{1}{5}$ s (1, 2, 3)

(ii) $\frac{2}{7}$ is two ...... s. $\left(\frac{1}{2}, \frac{1}{7}, \frac{1}{5}\right)$

(iii) Five $\frac{1}{6}$ s is equal to .... $\left(\frac{1}{30}, \frac{5}{6}, \frac{1}{5}\right)$

(iv) $\frac{\boxed{...}}{12}$ is equivalent to $\frac{2}{3}$. (2 , 4, 8)

(3) Write down two equivalent fractions for each of the following fractions.

(i) $\frac{2}{3}$ (ii) $\frac{3}{5}$ (iii) $\frac{6}{8}$ (iv) $\frac{36}{48}$

(4) For each of the following fractions, write down the equivalent fraction with the smallest denominator.

$\frac{18}{30}, \frac{16}{24}, \frac{10}{35}$

(5) Write down the fractions $\frac{4}{7}, \frac{1}{7}, \frac{6}{7}, \frac{5}{7}$ in ascending order.

(6) Write down the fractions $\frac{7}{12}, \frac{5}{12}, \frac{2}{3}, \frac{1}{4}$ in descending order.

(7) If Sithmi obtained 21 marks out of a total of 25 marks for a test, express her marks as a fraction of the total marks.

(8) A vendor bought a stock of 50 mangoes of which 8 were spoilt.
   (i) Express the number of spoilt fruits as a fraction of the total number of fruits.
   (ii) Express the number of good fruits as a fraction of the total number of fruits.

10.2 Mixed numbers and improper fractions


1


$\frac{1}{2}$

A whole cake and exactly half of an identical cake are shown in the figure. When the whole cake is taken as a unit, it is expressed as 1 and the other part which is exactly half of such a cake, is expressed as $\frac{1}{2}$. Therefore, the total amount of cake in the figure is $1+\frac{1}{2}$ times the whole cake. This is written as $1\frac{1}{2}$, and read as **one and a half**.

When a number which is the sum of a whole number and a proper fraction is written in this manner, it is defined as a mixed number. The whole number in the mixed number is called the whole number part and the proper fraction in it is called the fractional part.

$1\frac{1}{2}$, $1\frac{7}{8}$, $2\frac{2}{5}$ and $3\frac{1}{3}$ are examples of mixed numbers. The whole number part of the mixed number $2\frac{2}{5}$ is 2, and its fractional part is $\frac{2}{5}$.

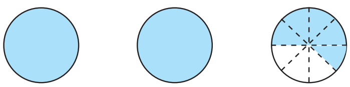
Let us write the mixed numbers represented in the following figures in the above manner.

1 $\quad \frac{2}{3}$

$$1 + \frac{2}{3} = 1\frac{2}{3}$$

1 $\quad \frac{1}{4}$

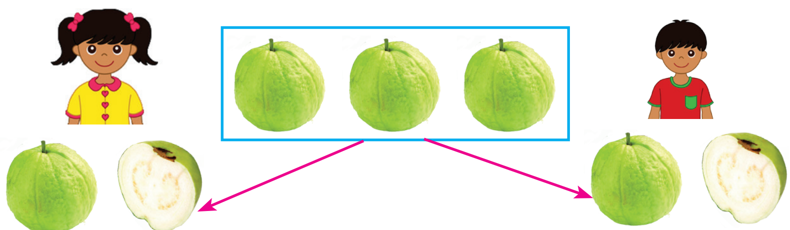
$$1 + \frac{1}{4} = 1\frac{1}{4}$$



1 $\quad$ 1 $\quad \frac{5}{8}$

$$1 + 1 + \frac{5}{8} = 2 + \frac{5}{8} = 2\frac{5}{8}$$

Now let us consider one way of dividing three guavas of the same size equally between two children.

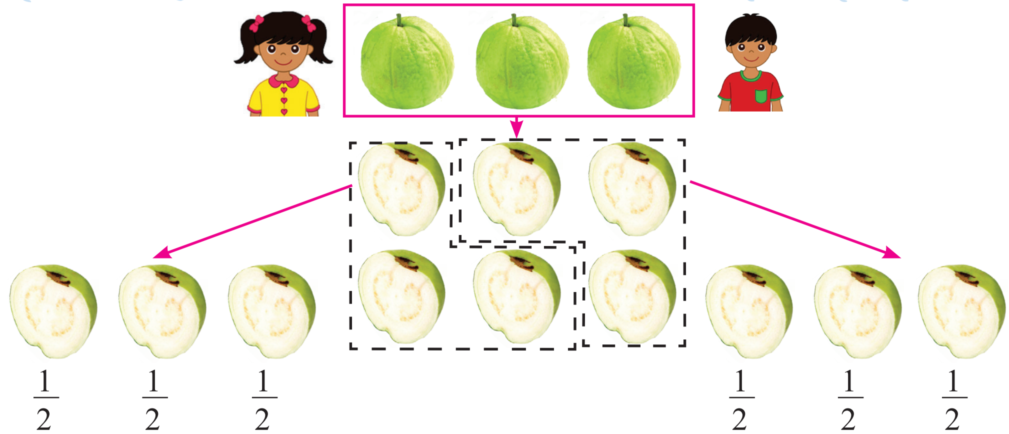


1 $\quad \frac{1}{2}$ $\qquad$ 1 $\quad \frac{1}{2}$

Here, each child receives one whole fruit and half of another fruit.

That is, the total amount that each child receives = $1+\frac{1}{2}$ fruits. This is written as $1\frac{1}{2}$.

Next, let us consider another way of dividing these three guavas equally between the two children.



3 halves $= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$.

(Divide each fruit into two equal parts)

3 halves $= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$.

Accordingly, each child gets 3 halves, which is an amount of $\frac{3}{2}$ guavas.

In both the above cases, each child receives the same amount of guava. Therefore, $\frac{3}{2} = 1\frac{1}{2}$.

The numerator of $\frac{3}{2}$ is greater than the denominator.

If the numerator of a fraction is greater or equal to the denominator, it is defined as an improper fraction.

We observed above that when three guavas are divided equally between two children, each child receives an amount of $\frac{3}{2}$ guavas. Therefore, $\frac{3}{2}$ represents the same value that is obtained when $3 \div 2$. That is, any proper fraction or improper fraction represents the number that is obtained when its numerator is divided by its denominator.

Example:: $\frac{2}{5} = 2 \div 5$ $\quad \frac{11}{3} = 11 \div 3$

$\frac{5}{2}$, $\frac{6}{3}$, $\frac{7}{5}$ and $\frac{11}{4}$ more examples of improper fractions.

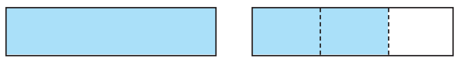
When the whole numbers 1, 2, 3 are expressed as $\frac{2}{2}$, $\frac{6}{3}$, and $\frac{15}{5}$ respectively, they too are considered as improper fractions.

Note that fractions with the numerator and denominator equal to each other are also considered as improper fractions.

10.3 Representing a mixed number as an improper fraction

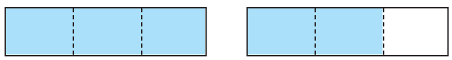
Let us find the shaded region in the figure using two methods.

First method



$1 + \frac{2}{3} = 1\frac{2}{3}$

Second method



The shaded region is five $\frac{1}{3}$s.

1 is equal to three $\frac{1}{3}$ s.

Five $\frac{1}{3}$ s. $= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{5}{3}$.

According to the above discussion, $1\frac{2}{3} = \frac{5}{3}$.

That is, the mixed number $1\frac{2}{3}$ can also be expressed as the improper fraction $\frac{5}{3}$.

Let us consider the mixed number $1\frac{3}{5}$.

Let us write the mixed number $1\frac{3}{5}$ as an improper fraction.

$$\begin{aligned} 1\frac{3}{5} &= 1 + \frac{3}{5} \\ &= \frac{5}{5} + \frac{3}{5} \\ &= \frac{8}{5} \end{aligned}$$

Example 1	**Example 2**
Express $2\frac{3}{4}$ as an improper fraction.	Express $3\frac{1}{2}$ as an improper fraction.
$2\frac{3}{4} = 1 + 1 + \frac{3}{4}$	$3\frac{1}{2} = 1 + 1 + 1 + \frac{1}{2}$
$= \frac{4}{4} + \frac{4}{4} + \frac{3}{4}$	$= \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{1}{2}$
$= \frac{4+4+3}{4}$	$= \frac{2+2+2+1}{2}$
$= \frac{11}{4}$	$= \frac{7}{2}$

Let us now consider an easy method of expressing a mixed number as an improper fraction. Let us consider the mixed number $1\frac{3}{5}$.

$$1 + \frac{3}{5} = \frac{5}{5} + \frac{3}{5}$$

$$= \frac{5+3}{5}$$

$$= \frac{(1 \times 5) + 3}{5} = \frac{8}{5}$$

- Multiply the whole number part in the mixed number, by the denominator of the fractional part and add this to the numerator of the fractional part.
- The value that is obtained is the value of the numerator of the improper fraction which is equal to the given mixed number.
- The denominator of the improper fraction is the same as the denominator of the fractional part of the mixed number.

Let us consider the following examples.

$$2\frac{3}{4} = \frac{(2 \times 4) + 3}{4} = \frac{8 + 3}{4} = \frac{11}{4}$$

$$3\frac{1}{2} = \frac{(3 \times 2) + 1}{2} = \frac{6 + 1}{2} = \frac{7}{2}$$

This process can be done mentally in one step; $7\frac{3}{8} = \frac{59}{8}$.

10.4 Expressing an improper fraction as a mixed number

Let us express $\frac{5}{3}$ as a mixed number .

Method I

$$\frac{5}{3} = \frac{3 + 2}{3}$$

$$= \frac{3}{3} + \frac{2}{3}$$

$$= 1 + \frac{2}{3} = 1\frac{2}{3}$$

Method II

$$\frac{5}{3} = 5 \div 3$$

$$\begin{array}{r|l} & 1 \\ 3 & 5 \\ & 3 \\ \hline & 2 \end{array}$$

The quotient of 5÷3 is 1 and the remainder is 2.

Let us write the above quotient as the whole number part of the solution. The remainder is the numerator of the fractional part. The denominator is the same as that of the improper fraction.

$$\therefore \frac{5}{3} = 1 + \frac{2}{3} = 1\frac{2}{3}$$

Example 1

Express $\frac{17}{10}$ as a mixed number.

Method I

$$\frac{17}{10} = \frac{10+7}{10}$$

$$= \frac{10}{10} + \frac{7}{10}$$

$$= 1\frac{7}{10}$$

Method II

$$\frac{17}{10} = 17 \div 10 = 1 + \frac{7}{10}$$

$$= 1\frac{7}{10}$$

$$\begin{array}{r|l} & 1 \\ \hline 10 & 17 \\ & 10 \\ \hline & 7 \end{array}$$

Example 2

Express $\frac{17}{4}$ as a mixed number.

Method I

$$\frac{17}{4} = \frac{4+4+4+4+1}{4}$$

$$= \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{1}{4}$$

$$= 1 + 1 + 1 + 1 + \frac{1}{4}$$

$$\frac{17}{4} = 4\frac{1}{4}$$

Method II

$$\frac{17}{4} = 17 \div 4 = 4 + \frac{1}{4}$$

$$= 4\frac{1}{4}$$

$$\begin{array}{r|l} & 4 \\ \hline 4 & 17 \\ & 16 \\ \hline & 1 \end{array}$$

Exercise 10.1

(1) Of the fractions given below, choose and write down the improper fractions.

(i) $\frac{8}{6}$, $\frac{49}{50}$, $\frac{31}{30}$, $\frac{19}{3}$, $\frac{3}{4}$

(2) Express each of the following mixed numbers as an improper fraction.

(i) $1\frac{1}{4}$ (ii) $2\frac{3}{5}$ (iii) $3\frac{1}{3}$ (iv) $7\frac{5}{8}$

(3) Express each of the following improper fractions as a mixed number.

(i) $\frac{14}{3}$ (ii) $\frac{13}{5}$ (iii) $\frac{26}{3}$ (iv) $\frac{94}{9}$

(4) Write down as a mixed number and as an improper fraction, the amount of guava that each child receives when 23 equal sized guavas are divided equally among 5 children.

10.5 Comparison of fractions

• Comparison of fractions having the same numerator

You have learnt that when two fractions with equal numerators are considered, the fraction with the smaller denominator is greater than the other fraction.

Accordingly, $\frac{4}{5}$ is greater than $\frac{4}{7}$. That is, $\frac{4}{5} > \frac{4}{7}$.

Further, when $\frac{5}{7}, \frac{5}{9}, \frac{5}{8}$ are arranged in ascending order we obtain

$\frac{5}{9}, \frac{5}{8}, \frac{5}{7}$. That is, $\frac{5}{9} < \frac{5}{8} < \frac{5}{7}$.

• Comparison of fractions having the same denominator

You have also learnt that when two fractions with equal denominators are considered, the fraction with the larger numerator is greater than the other fraction.

Accordingly, $\frac{3}{5}$ is greater than $\frac{2}{5}$. That is, $\frac{2}{5} < \frac{3}{5}$.

Further, when $\frac{9}{11}, \frac{2}{11}, \frac{15}{11}$ are arranged in ascending order we obtain $\frac{2}{11}, \frac{9}{11}, \frac{15}{11}$.

That is, $\frac{2}{11} < \frac{9}{11} < \frac{15}{11}$.

Further comparison of fractions

Now let us consider how the greater fraction is identified when two fractions with unequal numerators and unequal denominators are compared by writing equivalent fractions having a common denominator.

Let us compare the fractions $\frac{5}{3}$ and $\frac{7}{6}$.

Let us find the fraction with denominator 6 that is equivalent to $\frac{5}{3}$.

To do this, let us multiply the numerator and denominator of $\frac{5}{3}$ by 2.

$$\frac{5}{3} = \frac{5 \times 2}{3 \times 2} = \frac{10}{6}$$

$$\frac{10}{6} > \frac{7}{6}.$$

Since $\frac{10}{6} = \frac{5}{3}$ we obtain $\frac{5}{3} > \frac{7}{6}$.

Therefore, of the two fractions $\frac{5}{3}$ and $\frac{7}{6}$, the larger fraction is $\frac{5}{3}$.

Let us compare the fraction $\frac{7}{12}$ and $\frac{5}{8}$.

Neither denominator of the fractions $\frac{7}{12}$ and $\frac{5}{8}$ can be written as a multiple of the other. In such situations, the fractions have to be converted into equivalent fractions that have a denominator which is a multiple of the denominators of both fractions. It is convenient to take the least common multiple (LCM) of 12 and 8 in this situation.

$$\begin{array}{r|l} 2 & 12,\ 8 \\ \hline 2 & 6,\ 4 \\ \hline & 3,\ 2 \end{array}$$

The least common multiple of 12 and 18 $\Big\}$ $= 2 \times 2 \times 3 \times 2$

$= 24$

$$\frac{7 \times 2}{12 \times 2} = \frac{14}{24}$$

$$\frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

$\frac{15}{24} > \frac{14}{24}$ Therefore $\frac{5}{8} > \frac{7}{12}$.

Example 1

Compare the fractions $\frac{17}{12}$ and $\frac{9}{5}$.

There is no number other than 1 which divides both 12 and 5 without remainder.

Therefore, the least common multiple of 12 and 5 $= 12 \times 5 = 60$.

$$\frac{17}{12} = \frac{17 \times 5}{12 \times 5} = \frac{85}{60}$$

$$\frac{9}{5} = \frac{9 \times 12}{5 \times 12} = \frac{108}{60}$$

Since $\frac{108}{60} > \frac{85}{60}$ we obtain, $\frac{9}{5} > \frac{17}{12}$.

A proper fraction is always smaller than an improper fraction with the same denominator.

10.6 Comparison of mixed numbers

- **Mixed numbers with unequal whole number parts**

Let us find the larger number from the mixed numbers $1\frac{1}{2}$ and $3\frac{2}{5}$.

- First, let us examine the whole number parts of the two mixed numbers.
- If the whole number parts are unequal, then the mixed number with the greater whole number part is the larger mixed number.

Accordingly, when the whole number parts of $1\frac{1}{2}$ and $3\frac{2}{5}$ are considered, they are 1 and 3 respectively. Since $3 > 1$, the larger mixed number is $3\frac{2}{5}$.

$3\frac{2}{5} > 1\frac{1}{2}$.

• Mixed numbers with equal whole number parts

Select the larger number from $3\frac{2}{5}$ and $3\frac{1}{2}$.

Method I

- The whole number parts of the above two numbers are equal.
- Therefore, let us compare the fractional parts of these mixed numbers.

Accordingly, let us compare the fractional parts $\frac{2}{5}$ and $\frac{1}{2}$ of the mixed numbers $3\frac{2}{5}$ and $3\frac{1}{2}$.

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$$

$$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$

Since $\frac{5}{10} > \frac{4}{10}$, we obtain $\frac{1}{2} > \frac{2}{5}$.

Therefore, $3\frac{1}{2} > 3\frac{2}{5}$.

Method II

- Express the mixed numbers as improper fractions.
- The larger mixed number can be selected by considering which equivalent improper fraction is larger.

$$3\frac{2}{5} = \frac{17}{5}$$

$$3\frac{1}{2} = \frac{7}{2}$$

Now let us write $\frac{17}{5}$ and $\frac{7}{2}$ as fractions with equal denominators.

$$\frac{17}{5} = \frac{17 \times 2}{5 \times 2} = \frac{34}{10}$$

$$\frac{7}{2} = \frac{7 \times 5}{2 \times 5} = \frac{35}{10}$$

Since $\frac{35}{10} > \frac{34}{10}$, we obtain $\frac{7}{2} > \frac{17}{5}$.

That is, $3\frac{1}{2} > 3\frac{2}{5}$.

Exercise 10.2

(1) For each of the following parts, select and write down the larger/largest fraction from the given fractions.

(i) $\frac{1}{3}, \frac{1}{5}$ (ii) $\frac{13}{7}, \frac{15}{7}$ (iii) $\frac{5}{11}, \frac{8}{11}, \frac{12}{11}$ (iv) $\frac{11}{3}, \frac{11}{7}, \frac{11}{5}$

(v) $\frac{7}{10}, \frac{4}{5}$ (vi) $\frac{3}{2}, \frac{5}{4}$ (vii) $\frac{3}{4}, \frac{2}{3}$ (viii) $\frac{15}{8}, \frac{7}{3}$

(2) For each of the following parts, select and write down the larger number from the given pair of mixed numbers.

(i) $3\frac{1}{4}, 7\frac{2}{3}$ (ii) $6\frac{2}{5}, 4\frac{1}{2}$ (iii) $5\frac{3}{8}, 5\frac{7}{8}$ (iv) $2\frac{4}{5}, 2\frac{4}{7}$

(v) $6\frac{1}{4}, 6\frac{3}{8}$ (vi) $1\frac{3}{4}, 1\frac{2}{3}$ (vii) $7\frac{5}{6}, 7\frac{4}{5}$ (viii) $6\frac{3}{7}, 6\frac{1}{5}$

(3) Fill in the blanks with the suitable symbol from <, > and =.

(i) $\frac{3}{7}$ .... $\frac{3}{5}$ (ii) $\frac{17}{9}$ ..... $\frac{15}{9}$ (iii) $\frac{25}{8}$ .... $\frac{13}{4}$ (iv) $\frac{4}{5}$ .... $\frac{2}{3}$

(v) $2\frac{1}{6}$ ..... $5\frac{1}{3}$ (vi) $7\frac{1}{2}$ ..... $3\frac{4}{5}$ (vii) $2\frac{1}{5}$ ..... $2\frac{2}{10}$

(viii) $4\frac{2}{3}$ ..... $4\frac{1}{2}$ (ix) $7\frac{3}{8}$ ..... $7\frac{1}{3}$

(4) A person divides a 10 acre land he owns into three equal portions and gives each of his sons a portion. He also divides a 15 acre land he owns into four equal portions and gives each of his daughters a portion. Find out whether the portion a son receives is larger or smaller than the portion a daughter receives. .

(5) The depth of the drain cut by three labourers A, B and C during a day are $1\frac{1}{4}$ m, $2\frac{3}{4}$ m and 2 m respectively. Which labourer has cut the drain with the least depth? Explain your answer.

Summary

- Fractions with the numerator greater or equal to the denominator are defined as improper fractions.
- Numbers which consist of a whole number part and a fractional part are defined as mixed numbers.
- Mixed numbers can be compared by first converting them into equivalent improper fractions.

10 Fractions (Part II)

By studying this lesson you will be able to
- add and subtract fractions.

10.7 Addition of fractions

• Addition of fractions having the same denominator

In Grade 6 you learnt to add proper fractions with equal denominators as well as proper fractions with unequal denominators.

Let us consider the addition of fractions with equal denominators.

$$\frac{2}{8} + \frac{9}{8} = \frac{2+9}{8} = \frac{11}{8}$$

When fractions with equal denominators are added, the denominator of the answer is the same as the denominators of the fractions that are added. The numerator of the answer is the sum of the numerators of the fractions that are added.

The above answer $\frac{11}{8}$ can also be expressed as a mixed number. Then the answer is $1\frac{3}{8}$.

• Addition of fractions with unequal denominators

When fractions with unequal denominators are being added, the given fractions need to be first converted into equivalent fractions with equal denominators, and then added.

Here, it is convenient to convert the given fractions into equivalent fractions that have the least common multiple of the denominators of the given fractions as the denominator.

Find the value of $\frac{7}{10} + \frac{7}{15}$.

The relationship between the denominators of $\frac{7}{10}$ and $\frac{7}{15}$ cannot be identified easily.

In such situations, where the denominators are not related, the given fractions need to be converted into equivalent fractions which have as their denominators a common multiple of the denominators of the given fractions.

Here is it convenient to select the least common multiple of 10 and 15.

$$\begin{array}{r|l} 5 & 10,\ 15 \\ \hline & 2,\ 3 \end{array}$$

The least common multiple of 10 and 15 $= 5 \times 2 \times 3$

$= 30$

$$\frac{7}{10} = \frac{7 \times 3}{10 \times 3} = \frac{21}{30}$$

$$\frac{7}{15} = \frac{7 \times 2}{15 \times 2} = \frac{14}{30}$$

$$\frac{7}{10} + \frac{7}{15} = \frac{21}{30} + \frac{14}{30} = \frac{35}{30} = \frac{7}{6} = 1\frac{1}{6}$$

Example 1

Find the value of $\frac{3}{2} + \frac{3}{8}$.

$$\frac{3}{2} + \frac{3}{8} = \frac{3 \times 4}{2 \times 4} + \frac{3}{8}$$

$$= \frac{12}{8} + \frac{3}{8}$$

$$= \frac{12 + 3}{8}$$

$$= \frac{15}{8}$$

$$= 1\frac{7}{8}$$

Example 2

Find the value of $\frac{1}{4} + \frac{2}{5}$.

Here it is convenient to take the least common multiple of 4 and 5 as the denominator each of the equivalent fractions. The least common multiple of 4 and 5 is 20.

$$\frac{1}{4} = \frac{1 \times 5}{4 \times 5} = \frac{5}{20}$$

$$\frac{2}{5} = \frac{2 \times 4}{5 \times 4} = \frac{8}{20}$$

$$\frac{1}{4} + \frac{2}{5} = \frac{5}{20} + \frac{8}{20}$$

$$= \frac{13}{20}$$

Example 3

Find the value of $\frac{17}{12} + \frac{9}{8}$.

The least common multiple of 12 and 8 is 24.

$$\frac{17}{12} + \frac{9}{8} = \frac{34}{24} + \frac{27}{24}$$
$$= \frac{61}{24}$$
$$= 2\frac{13}{24}$$

Example 4

Find the value of $\frac{5}{3} + \frac{3}{8} + \frac{7}{4}$.

The least common multiple of 3, 8 and 4 is 24.

$$\frac{5}{3} + \frac{3}{8} + \frac{7}{4} = \frac{40}{24} + \frac{9}{24} + \frac{42}{24}$$
$$= \frac{91}{24}$$
$$= 3\frac{19}{24}$$

- When adding fractions, some of the steps given in the above examples can be done mentally and the answer can be obtained in a few steps.
- When fractions are simplified, if the answer obtained is a proper fraction, it should be given in its simplest form, and if it is an improper fraction, it should be written as a mixed number.

Exercise 10.3

(1) Evaluate the following.

(i) $\frac{2}{9} + \frac{7}{9} + \frac{5}{9}$ (ii) $\frac{13}{11} + \frac{4}{11}$ (iii) $\frac{7}{6} + \frac{13}{12}$ (iv) $\frac{2}{7} + \frac{3}{5}$

(v) $\frac{12}{5} + \frac{1}{3} + \frac{2}{15}$ (vi) $\frac{13}{4} + \frac{2}{5}$ (vii) $\frac{3}{2} + \frac{5}{4} + \frac{4}{3}$

• Addition of mixed numbers

Let us consider how the two mixed numbers $1\frac{2}{5}$ and $1\frac{1}{5}$ are added together. This is written as $1\frac{2}{5} + 1\frac{1}{5}$.

Method I

The whole number parts and the fractional parts can be added separately.

$$1\frac{2}{5} + 1\frac{1}{5} = 1 + 1 + \frac{2}{5} + \frac{1}{5}$$
$$= 2 + \frac{2+1}{5}$$
$$= 2 + \frac{3}{5}$$
$$= 2\frac{3}{5}$$

Method II

The mixed numbers can be written as improper fractions and added.

$1\frac{2}{5} = \frac{7}{5}$ and $1\frac{1}{5} = \frac{6}{5}$

$$1\frac{2}{5} + 1\frac{1}{5} = \frac{7}{5} + \frac{6}{5}$$
$$= \frac{7+6}{5}$$
$$= \frac{13}{5}$$
$$= 2\frac{3}{5}$$

Method I is more suitable.

Example 1

Find the value of $2\frac{3}{7} + \frac{2}{7}$.

$$2\frac{3}{7} + \frac{2}{7} = 2 + \frac{3}{7} + \frac{2}{7}$$
$$= 2 + \frac{5}{7}$$
$$= 2\frac{5}{7}$$

Example 2

Find the value of $1\frac{1}{3} + 2\frac{5}{12}$.

$$1\frac{1}{3} + 2\frac{5}{12} = (1+2) + \left(\frac{1}{3} + \frac{5}{12}\right)$$
$$= 3 + \left(\frac{1 \times 4}{3 \times 4} + \frac{5}{12}\right)$$
$$= 3 + \left(\frac{4}{12} + \frac{5}{12}\right)$$
$$= 3 + \frac{9}{12} = 3\frac{9}{12} = 3\frac{3}{4}$$

Example 3

Find the value of $2\frac{2}{3} + \frac{1}{4}$.

$$2\frac{2}{3} + \frac{1}{4} = 2 + \left(\frac{2}{3} + \frac{1}{4}\right)$$
$$= 2 + \left(\frac{8}{12} + \frac{3}{12}\right)$$
$$= 2 + \left(\frac{8+3}{12}\right)$$
$$= 2 + \frac{11}{12}$$
$$= 2\frac{11}{12}$$

Example 4

Find the value of $2\frac{1}{5} + 4\frac{2}{3}$.

$$2\frac{1}{5} + 4\frac{2}{3} = (2+4) + \left(\frac{1}{5} + \frac{2}{3}\right)$$
$$= 6 + \left(\frac{3}{15} + \frac{10}{15}\right)$$
$$= 6 + \left(\frac{3+10}{15}\right)$$
$$= 6 + \frac{13}{15}$$
$$= 6\frac{13}{15}$$

Example 5

Find the value of $1\frac{2}{3} + 2\frac{3}{5} + \frac{5}{6}$.

$$1\frac{2}{3} + 2\frac{3}{5} + \frac{5}{6} = (1+2) + \left(\frac{2}{3} + \frac{3}{5} + \frac{5}{6}\right)$$
$$= 3 + \left(\frac{20}{30} + \frac{18}{30} + \frac{25}{30}\right) = 3 + \frac{63}{30} = 3 + \frac{63 \div 3}{30 \div 3}$$
$$= 3 + \frac{21}{10}$$
$$= 3 + 2\frac{1}{10}$$
$$= 5\frac{1}{10}$$

Exercise 10.4

(1) Evaluate the following.

(i) $3\frac{2}{7} + \frac{3}{7}$ (ii) $2\frac{4}{10} + 3\frac{3}{10}$ (iii) $1\frac{1}{9} + 2\frac{2}{9} + \frac{4}{9}$

(iv) $2\frac{1}{3} + 3\frac{5}{9}$ (v) $\frac{7}{12} + 2\frac{1}{3}$ (vi) $4\frac{3}{5} + 2\frac{1}{10}$

(vii) $2\frac{1}{4} + \frac{2}{3}$ (viii) $5\frac{2}{3} + 3\frac{2}{5}$ (ix) $2\frac{2}{7} + 1\frac{3}{4}$

(x) $4\frac{3}{10} + 3\frac{1}{4}$ (xi) $5\frac{2}{5} + 2\frac{3}{7}$ (xii) $2\frac{7}{12} + 3\frac{5}{8}$

(xiii) $1\frac{2}{3} + 2\frac{1}{3} + 2\frac{5}{6}$ (xiv) $3\frac{1}{4} + 1\frac{1}{2} + \frac{1}{6}$

(xv) $3\frac{5}{6} + 2\frac{3}{4} + 5\frac{1}{3}$

(2) A seamstress states that $1\frac{1}{6}$ m of material is required for a shirt and $2\frac{3}{4}$ m of material is required for a dress. Find the amount of material of a certain type that is required for a shirt and a dress.

(3) A farmer has cultivated paddy in an area of $3\frac{1}{2}$ square kilometres and vegetables in an area of $1\frac{2}{5}$ square kilometres. Find the total cultivated area.

10.8 Subtraction of fractions

Let us now, using examples, describe how to subtract fractions with equal denominators as well as fractions with unequal denominators.

When fractions with unequal denominators are being subtracted, the given fractions are first converted into equivalent fractions with equal denominators and then subtracted.

Example 1

Find the value of $\frac{7}{5} - \frac{1}{5}$.

$$\frac{7}{5} - \frac{1}{5} = \frac{7-1}{5}$$
$$= \frac{6}{5}$$
$$= 1\frac{1}{5}$$

Example 2

Find the value of $\frac{17}{8} - \frac{3}{2}$.

$$\frac{17}{8} - \frac{3}{2} = \frac{17}{8} - \frac{3}{2}$$
$$= \frac{17}{8} - \frac{12}{8}$$
$$= \frac{17-12}{8}$$
$$= \frac{5}{8}$$

Example 3

Find the value of $\frac{1}{2} - \frac{1}{3}$.

$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}, \qquad \frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

$$\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{3-2}{6} = \frac{1}{6}$$

Exercise 10.5

(1) Find the value of each of the following.

(i) $\frac{8}{11} - \frac{7}{11}$ (ii) $\frac{13}{12} - \frac{7}{12}$ (iii) $\frac{3}{5} - \frac{1}{5}$ (iv) $\frac{19}{11} - \frac{8}{11}$

(v) $\frac{3}{4} - \frac{1}{2}$ (vi) $\frac{2}{3} - \frac{7}{12}$ (vii) $\frac{15}{7} - \frac{11}{14}$ (viii) $\frac{13}{10} - \frac{1}{2}$

(ix) $\frac{3}{2} - \frac{6}{5}$ (x) $\frac{5}{6} - \frac{3}{4}$ (xi) $\frac{11}{7} - \frac{4}{5}$

(xii) $\frac{9}{8} - \frac{5}{6}$ (xiii) $\frac{7}{8} - \frac{5}{12}$ (xiv) $\frac{8}{9} - \frac{5}{6}$

Subtracting mixed numbers

Mother had $3\frac{2}{3}$ m of material. She cut $1\frac{1}{3}$ m from this material to sew a dress for her daughter. The amount of material that is remaining can be written as follows.

Amount of material remaining $= 3\frac{2}{3} - 1\frac{1}{3}$.

Method I

In instances such as this, where mixed numbers are being subtracted, the whole number parts and the fractional parts can be simplified separately. Now let us consider how this is done.

$$3\frac{2}{3} - 1\frac{1}{3} = (3-1) + \frac{2}{3} - \frac{1}{3}$$

$$= 2 + \frac{2-1}{3}$$

$$= 2 + \frac{1}{3}$$

$$= 2\frac{1}{3}$$

Method II

This simplification can also be done by converting the mixed numbers into improper fractions. Let us now consider how this is done.

$$3\frac{2}{3} - 1\frac{1}{3} = \frac{11}{3} - \frac{4}{3}$$

$$= \frac{11-4}{3}$$

$$= \frac{7}{3}$$

$$= 2\frac{1}{3}$$

Example 1

Evaluate $2\frac{7}{9} - \frac{2}{9}$.

$$2\frac{7}{9} - \frac{2}{9} = 2 + \left(\frac{7}{9} - \frac{2}{9}\right)$$
$$= 2 + \left(\frac{7-2}{9}\right)$$
$$= 2 + \frac{5}{9}$$
$$= 2\frac{5}{9}$$

Example 2

Evaluate $6\frac{5}{9} - \frac{1}{3}$.

$$6\frac{5}{9} - \frac{1}{3} = 6 + \left(\frac{5}{9} - \frac{1}{3}\right)$$
$$= 6 + \left(\frac{5}{9} - \frac{1 \times 3}{3 \times 3}\right)$$
$$= 6 + \left(\frac{5}{9} - \frac{3}{9}\right)$$
$$= 6 + \frac{2}{9} = 6\frac{2}{9}$$

Example 3

Evaluate $3\frac{4}{5} - 2\frac{1}{5}$.

$$3\frac{4}{5} - 2\frac{1}{5} = (3-2) + \left(\frac{4}{5} - \frac{1}{5}\right)$$
$$= 1 + \left(\frac{4-1}{5}\right)$$
$$= 1\frac{3}{5}$$

Example 4

Evaluate $5\frac{7}{10} - 2\frac{2}{15}$.

$$5\frac{7}{10} - 2\frac{2}{15} = (5-2) + \left(\frac{7}{10} - \frac{2}{15}\right)$$
$$= 3 + \left(\frac{21}{30} - \frac{4}{30}\right)$$
$$= 3 + \frac{17}{30}$$
$$= 3\frac{17}{30}$$

Example 5

Evaluate $7\frac{2}{3} - \frac{1}{4}$.

$$7\frac{2}{3} - \frac{1}{4} = 7 + \left(\frac{2}{3} - \frac{1}{4}\right)$$

The LCM of 3 and 4 is 12.

$$7\frac{2}{3} - \frac{1}{4} = 7 + \left(\frac{2 \times 4}{3 \times 4} - \frac{1 \times 3}{4 \times 3}\right)$$
$$= 7 + \left(\frac{8}{12} - \frac{3}{12}\right)$$
$$= 7 + \frac{5}{12} = 7\frac{5}{12}$$

Example 6

Evaluate $3\frac{1}{5} - 2\frac{1}{10}$.

$$3\frac{1}{5} - 2\frac{1}{10} = (3-2) + \left(\frac{1}{5} - \frac{1}{10}\right)$$
$$= 1 + \left(\frac{1 \times 2}{5 \times 2} - \frac{1}{10}\right)$$
$$= 1 + \left(\frac{2}{10} - \frac{1}{10}\right)$$
$$= 1 + \frac{1}{10}$$
$$= 1\frac{1}{10}$$

Example 7

Evaluate $3\frac{2}{7} - 1\frac{1}{2}$.

Method I

$$3\frac{2}{7} - 1\frac{1}{2} = (3-1) + \left(\frac{2}{7} - \frac{1}{2}\right)$$
$$= 2 + \left(\frac{4}{14} - \frac{7}{14}\right)$$
$$= 2 + \frac{4-7}{14}$$
$$= 1 + 1 + \frac{4-7}{14}$$
$$= 1 + \frac{14}{14} + \frac{4-7}{14}$$
$$= 1 + \frac{14+4-7}{14} = 1 + \frac{11}{14}$$
$$= 1\frac{11}{14}$$

Method II

$$3\frac{2}{7} - 1\frac{1}{2} = \frac{23}{7} - \frac{3}{2}$$
$$= \frac{46}{14} - \frac{21}{14}$$
$$= \frac{25}{14}$$
$$= 1\frac{11}{14}$$

In such instances, it is easier to first convert the mixed numbers into improper fractions and then simplify them.

Exercise 10.6

(1) Evaluate the following.

(i) $2\frac{3}{5} - 1\frac{1}{5}$ (ii) $4\frac{5}{7} - 1\frac{4}{7}$ (iii) $2\frac{7}{8} - \frac{4}{8}$

(iv) $2 - 1\frac{1}{4}$ (v) $3 - 1\frac{5}{6}$ (vi) $2 - 1\frac{5}{16}$

(vii) $8\frac{7}{10} - 3\frac{2}{5}$ (viii) $2\frac{2}{5} - 1\frac{3}{20}$ (ix) $2\frac{2}{3} - 1\frac{1}{2}$

(x) $3\frac{3}{4} - 1\frac{7}{18}$ (xi) $6\frac{5}{8} - 4\frac{1}{6}$ (xii) $4\frac{3}{10} - 2\frac{4}{15}$

(2) Sachi travelled $3\frac{7}{10}$ kilometres to his brother Gamini's house by going the initial $3\frac{1}{2}$ kilometres by bus and walking the remaining distance. What was the distance that Sachi walked?

(3) A farmer owns a plot of land of area 4 hectares. He has cultivated kurakkan (finger millet) in $2\frac{1}{2}$ hectares. What is the extent of the land in which he has not cultivated kurakkan?

Miscellaneous Exercise

(1) (i) Express $7\frac{3}{5}$ as an improper fraction.

(ii) Express $\frac{50}{11}$ as a mixed number.

(2) (i) Write the fractions $1\frac{1}{4}, \frac{15}{7}, \frac{5}{3}, \frac{1}{2}$ in ascending order.

(ii) Write the fractions $2\frac{5}{3}, 7\frac{1}{3}, \frac{3}{4}, \frac{5}{7}$ in descending order.

(3) Evaluate the following.

(i) $\frac{1}{5} + 1\frac{1}{4} + 3\frac{5}{7}$ (ii) $\frac{3}{5} + 3\frac{5}{7} + 5\frac{1}{4}$ (iii) $7\frac{2}{3} - 4\frac{1}{4}$

(iv) $4\frac{5}{6} - 1\frac{3}{5}$ (v) $4\frac{5}{8} - 2\frac{1}{3}$ (vi) $2\frac{1}{2} - 1\frac{3}{4}$

(4) Malinga walked for three hours at $3\frac{1}{2}$ kilometres per hour. Find the total distance he walked during the three hours as an improper fraction.

Summary

- When fractions are simplified, if the answer obtained is a proper fraction, it should be given in its simplest form, and if it is an improper fraction, it should be written as a mixed number.