9 Angles

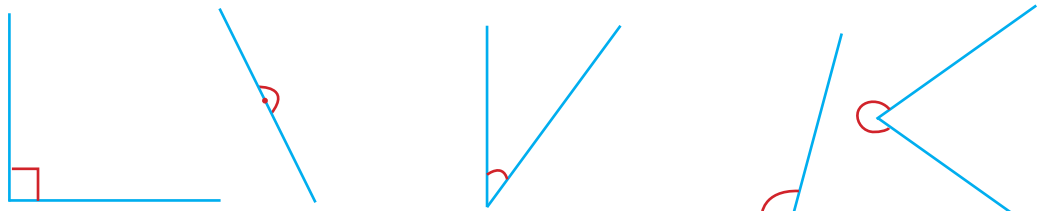
By studying this lesson you will be able to

- identify the dynamic or static nature of an angle,
- name angles,
- measure and draw angles using the protractor, and
- classify angles based on their magnitude.

9.1 Angles

You learnt in grade 6 that an angle is created when two straight line segments meet each other.

Below are a few types of angles we identified.



Right angle | Straight angle | Acute angle | Obtuse angle | Reflex angle

Do the following review exercise to recall the facts you have learnt about angles.

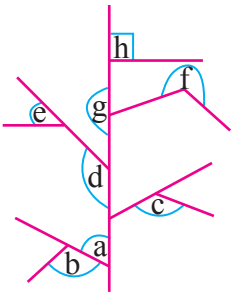
Review Exercise

(1) Choose the figures that are angles and write down the corresponding letters.

(a) (b) (c) (d)

(2) Identify the angles in the figure below and complete the table.

Angle	Type of Angle	Angle	Type of Angle
a		e	
b		f	
c		g	
d		h	

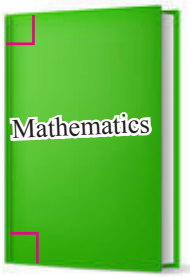


(3) Draw an angle of each type on a square ruled paper. Write the type of angle next to the corresponding figure.
Acute angle, Right angle, Obtuse angle, Straight angle, Reflex angle

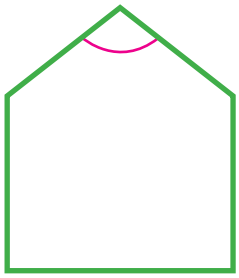
9.2 The dynamic or static nature of an angle

Let us investigate more on angles.

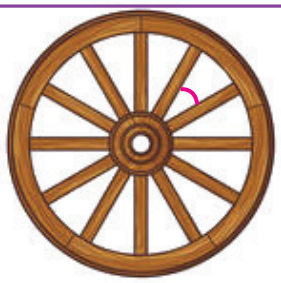
If we observe our surroundings, we can identify many angles. A few examples are given below.



Angle between the edges of a cover of a book



Angle between two parts of the roof.

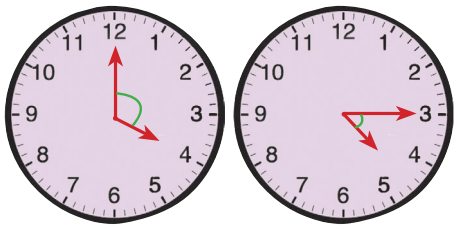


Angles between the adjacent spokes of a wheel

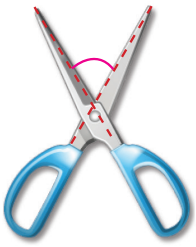
A common property of the above angles is that their magnitude does not change.

- If the magnitude of an angle does not change, then it is static in nature.
- So the angles in the above figures are static in nature.
- Note that the magnitude of the angle between two spokes of a wheel does not change, even when the wheel is turning.

Let us now consider some situations that involve rotation.



The angle between the hour hand and the minute hand of a clock changes in magnitude with time. The figure shows this angle at 4 p.m. and 4.15 p.m.



The angle between the two blades of a pair of scissors changes when it is used for cutting.

The angle between the top edge of a door and a door frame changes when the door is being opened or closed.

In the examples given above, let us consider the **arms** of the angle involved.

We see that there is a rotation of both arms or of one of them. Therefore, the magnitude of the angle changes. This is the **dynamic nature** of such an angle.

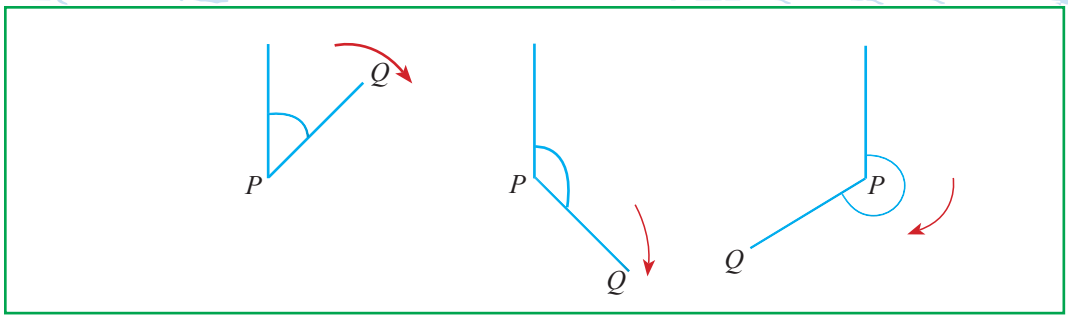
Let us understand the dynamic nature of an angle further by doing the following activity.

Activity 1

Step 1 - Take a fresh green ekel and bend it into two parts at the centre, taking care not to break it.

Step 2 - Overlap the two parts of the ekel and place it on a table. Hold one part tightly on the table.

Step 3 - In your exercise book, draw several situations that are obtained by rotating the other part on the table.
A few such situations that can be obtained are shown below.



- You can see that the magnitude of the angle between the two parts of the ekel changes. That is, this angle is dynamic in nature.
- When both parts of the ekel are rotated too the magnitude of the angle between the two parts changes.

A rotation which is in the same direction as the rotation of the arms of a clock is defined as a clockwise rotation. A rotation which is in the opposite direction to that of the rotation of the arms of a clock is defined as an anticlockwise rotation.

Exercise 9.1

(1) (i) Write down 3 instances where you can observe angles which are dynamic in nature in your surrounding environment.

(ii) Write down 3 instances where you can observe angles which are static in nature in your surrounding environment.

(2) (i) Give an example of an angle which is static in nature where the positions of the arms of the angle are fixed.

(ii) Give an example of an angle which is static in nature where there is a change in the positions of the arms of the angle.

(iii) Give an example of an angle which is dynamic in nature where there is a change in the position of only one arm of the angle.

(iv) Give an example of an angle which is dynamic in nature where there is a change in the positions of both arms of the angle.

9.3 Naming Angles

Let us now consider how angles are named.

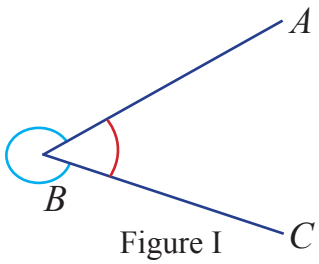
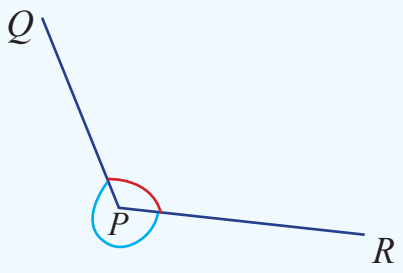


Figure I

- In Figure I, two angles have been created by the straight line segments AB and BC meeting.
- The straight line segments AB and BC are defined as the "arms of the angle". The point B where AB and BC meet is defined as the "vertex of the angle".
- The magnitude of the angle which is indicated in red is less than that of a straight angle; that is, less than the magnitude of two right angles.
- The magnitude of the angle indicated in blue is greater than that of a straight angle.
- The angle indicated in red is named as angle ABC and is written as , $A\hat{B}C$ or $C\hat{B}A$
- Here we write the letter which indicates the vertex in the middle and the other two letters beside it.
- The angle indicated in blue is named as the reflex angle ABC and is written as reflex angle $A\hat{B}C$ or reflex angle $C\hat{B}A$.
- In some books angle ABC is written as $\measuredangle ABC$.

Example 1

Draw the angles with the straight line segments PQ and PR as their arms. Name the two angles.



Since P is common to both arms, P is the vertex of the angles. Therefore the angle indicated in red is $Q\hat{P}R$ and the angle indicated in blue is the reflex angle $Q\hat{P}R$.

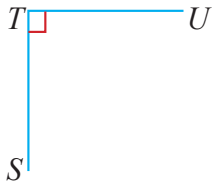
Example 2

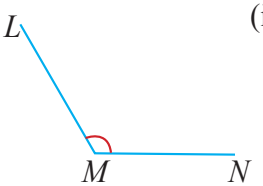
Write down the vertex and the arms of $D\hat{E}F$.

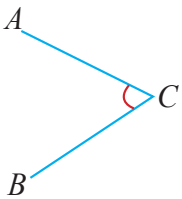
↳ Since the letter in the middle of $D\hat{E}F$ is E, the vertex of the angle is E and the arms of the angle are ED and EF.

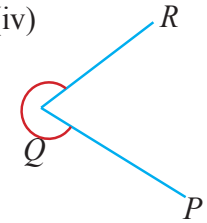
Exercise 9.2

(1) Write down the arms and the vertex of each of the angles given below.

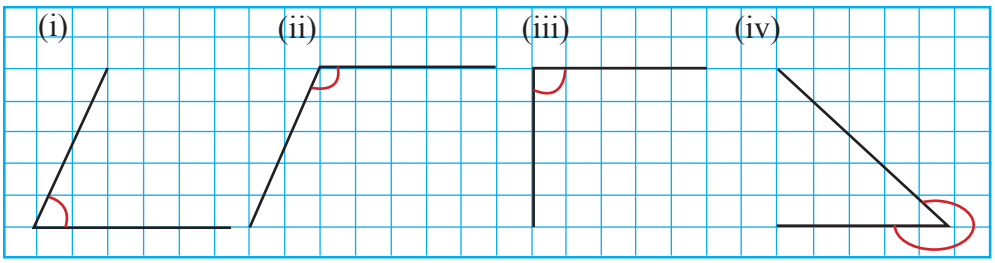
(i) 

(ii) 

(iii) 

(iv) 

(2) Copy each of the angles given below and name them using letters of the English alphabet.



(3) Draw and name an angle of your choice on a square ruled paper.

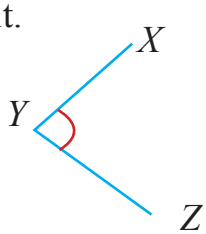
(4) Draw an obtuse angle with arms XY and YZ on a square ruled paper.

(5) Draw an angle and name it $D\hat{E}F$. Name its arms and its vertex.

(6) Draw a reflex angle and name it.

(7) Draw a right angle on a square ruled paper and name it.

(8) Prabath has written the angle in the figure as $X\hat{Y}Z$. Sumudu has written it as $Z\hat{Y}X$. Kasun says that both Prabath and Sumudu are correct. Do you agree with Kasun? Explain your answer.



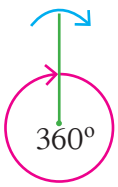
9.4 Measuring angles

There are standard units and instruments to measure distance, mass, time and the volume of a liquid. You learnt about these in grade 6.

Now let us learn about a standard unit and an instrument used to measure angles.

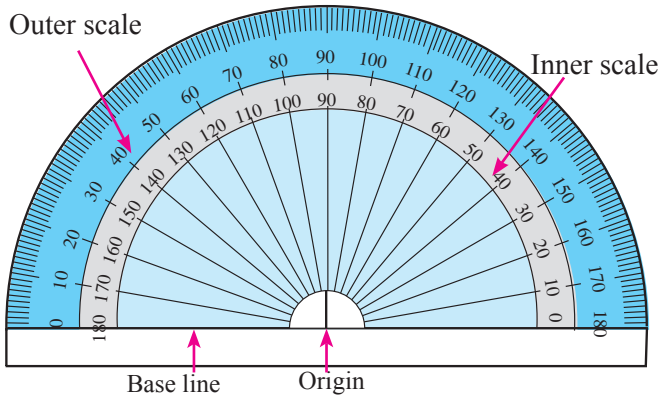
The standard unit used to measure angles is **degrees**. One degree is written as 1^{o}.

The angle that is formed when a straight line segment completes one full circle by rotating about a point is 360^{o}.



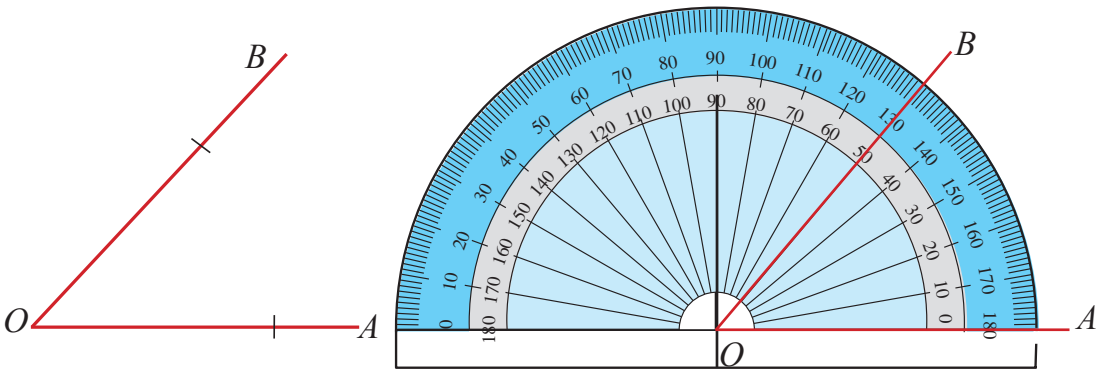
The instrument used to measure angles is made of one half of a full circle. It is called a "**protractor**". The figure of a protractor is shown below. It is numbered from 0^{o} to 180^{o} clockwise and anticlockwise. The line indicated by 0 - 0 is called the "base line".

There are two scales indicated in the protractor. They are the inner scale and the outer scale.



The long line segments on the outer scale are marked as 0, 10, 20, …, 180. The gap between every pair of long line segments is again divided into 10 similar parts using short line segments. As indicated in the figure, the magnitude of the angle between two long line segments is 10^{o}.

Let us now see how we can use the protractor to measure the angle $A\hat{O}B$ in the figure.



Place the protractor on the figure such that the origin and the base line coincide with the vertex O and the arm OA respectively.

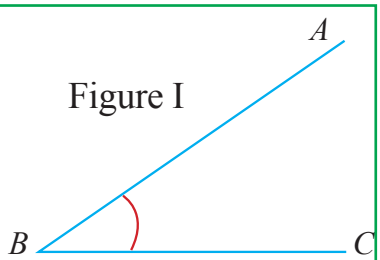
Then the arm OB coincides with the line indicated by 50^{o} in the inner scale (Note that OA coincides with 0^{o} on this scale). Therefore the magnitude of the angle $A\hat{O}B$ is 50^{o}, and we write $A\hat{O}B = 50^{o}$.

By observing this figure, we see that an angle of 1^{o} is a small angle which is difficult to draw.

Activity 2

Step 1 - In your exercise book, draw an angle similar to the one in Figure I using a ruler.

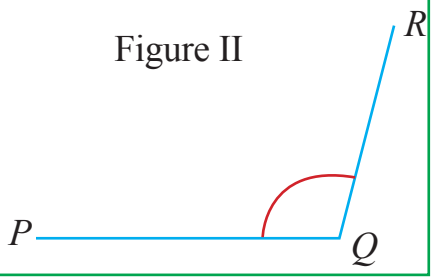
Figure I



Step 2 - Measure the magnitude of the angle drawn, and write it inside the space margined by AB, BC and the red arc.

Step 3 - Draw an angle similar to the one in Figure II below, measure the magnitude of the angle and write it down as done in step 2.

Figure II

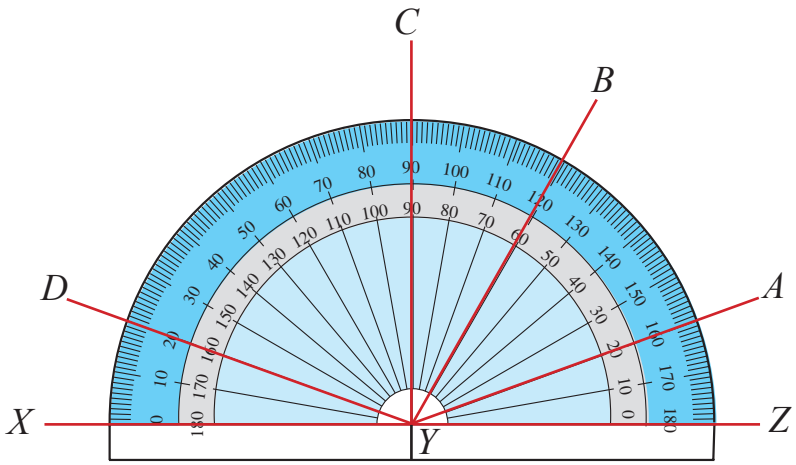


Exercise 9.3

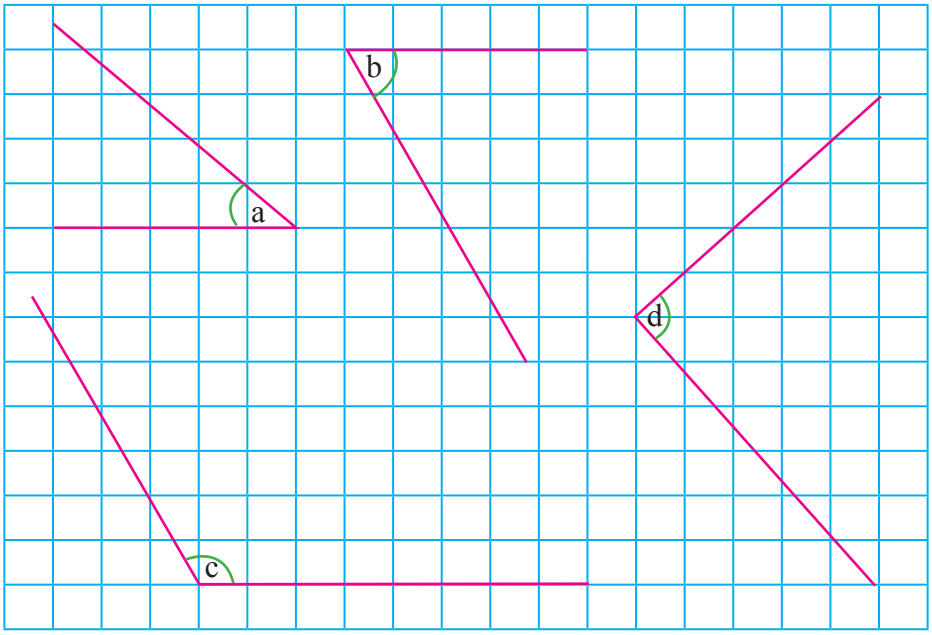
(1) Write down the magnitude of each angle using the given figure.

(i) $X\hat{Y}Z$ (ii) $Z\hat{Y}A$ (iii) $X\hat{Y}C$ (iv) $B\hat{Y}Z$

(v) $X\hat{Y}B$ (vi) $C\hat{Y}Z$ (vii) $X\hat{Y}A$ (viii) $Z\hat{Y}D$

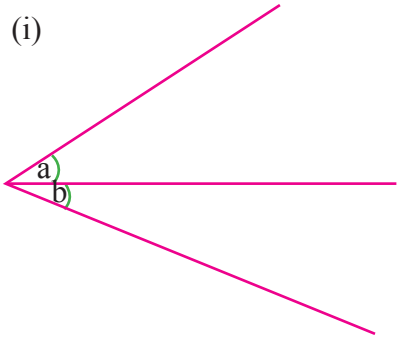


(2) Draw each of the angles below on a square ruled paper. Measure and write the magnitude of each angle.

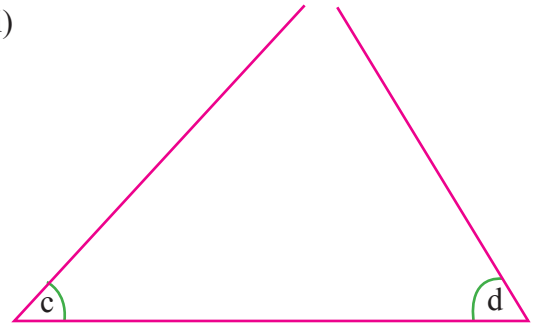


(3) Draw the following figures in your exercise book. Measure and write down the magnitude of each of the angles indicated by the English letters.

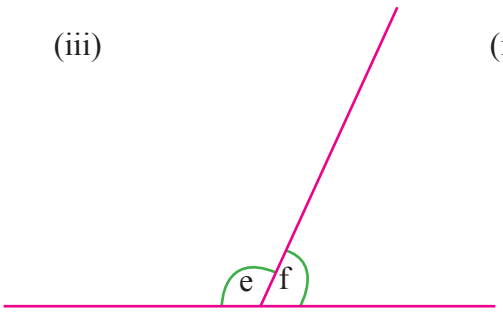
(i)



(ii)



(iii)



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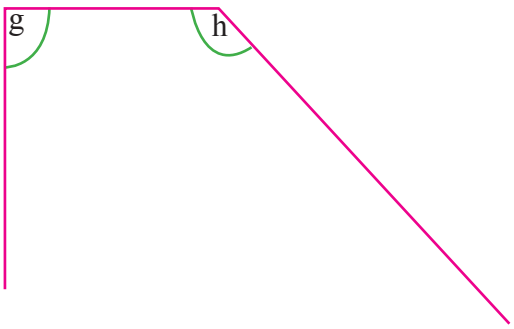


9.5 Drawing angles with given magnitude

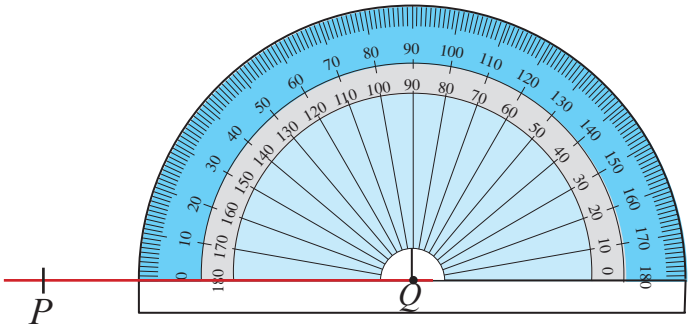
Let us now draw angles when their magnitude is given.

Activity 3

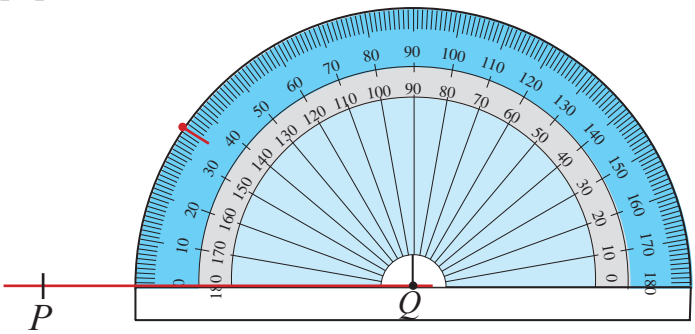
Performing the steps given below, draw the angle $P\hat{Q}R = 35^{o}$

Step 1 - Using the ruler, draw a straight line segment and name it PQ.

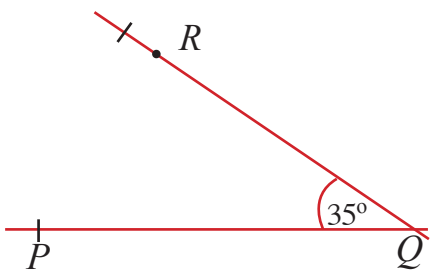
Step 2 - Since the vertex of the angle is Q, place the protractor so that its origin and the base line coincide with Q and PQ respectively.



Step 3 - Now find 35^{o} in the outer scale. Place a dot mark on the paper at 35^{0}.

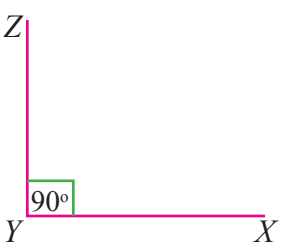


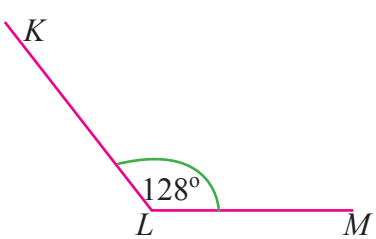
Step 4 - Remove the protractor. Name the dot marked in step 3 as R. Now draw a straight line from Q to R. Write the magnitude of the angle $P\hat{Q}R$ as 35^{0}.



As above, draw the following:

(i) $X\hat{Y}Z = 90^{o}$ (ii) $K\hat{L}M = 128^{o}$





Exercise 9.4

(1) Draw the following angles.

(i) $A\hat{B}C = 48^{\circ}$ (ii) $P\hat{Q}R = 90^{\circ}$ (iii) $K\hat{L}M = 130^{\circ}$ (iv) $X\hat{Y}Z = 28^{\circ}$

(2) (i) Draw a straight line segment and name it PQ.

(ii) Draw the arm PR such that $Q\hat{P}R = 82^{\circ}$.

(iii) Draw the arm QS such that $P\hat{Q}S = 43^{\circ}$.

(3) (i) Draw any triangle you like and name it ABC.

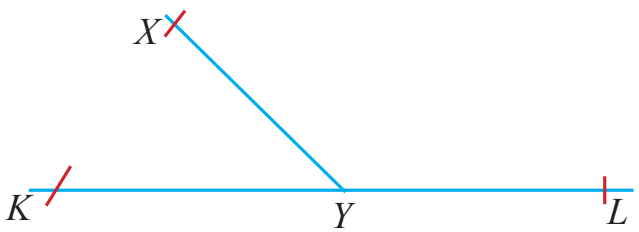
(ii) Measure $A\hat{B}C$, $B\hat{C}A$ and $C\hat{A}B$ and write their magnitudes separately.

(iii) Obtain the value of $A\hat{B}C + B\hat{C}A + C\hat{A}B$ using the measured values.

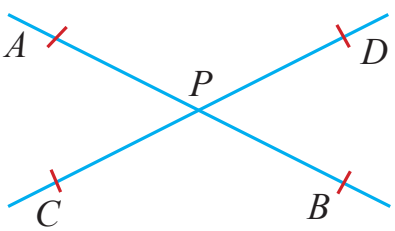
(4) (i) Draw two straight line segments KL and XY so that they meet each other at Y.

(ii) Measure and write down the magnitudes of $K\hat{Y}X$ and $X\hat{Y}L$

(iii) Obtain $K\hat{Y}X + X\hat{Y}L$.



(5) (i) As given in the figure, draw two straight line segments AB and CD so that they intersect each other.



(ii) Measure and write the magnitudes of $A\hat{P}C$, $C\hat{P}B$, $B\hat{P}D$ and $D\hat{P}A$ separately.

(iii) Write the relationship between $A\hat{P}C$ and $B\hat{P}D$.

(iv) Write the relationship between $A\hat{P}D$ and $C\hat{P}B$.

(6) Dasun says that the angle in Figure (II) is larger than the angle in Figure (I). Do you agree? Explain your answer.

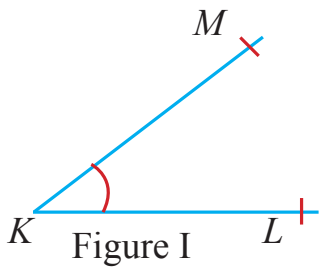


Figure I

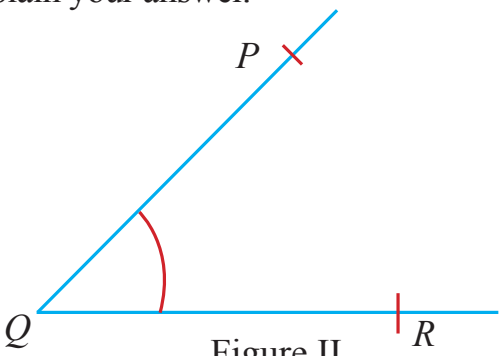


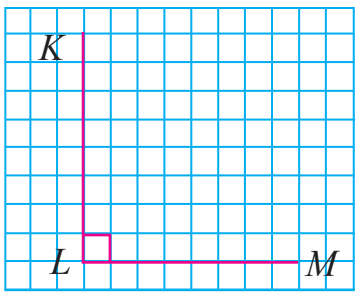
Figure II

9.6 Classification of angles

We learnt in grade 6 to classify angles using a right angle. Magnitude of a right angle is 90°. We can classify angles by comparing them with 90°.

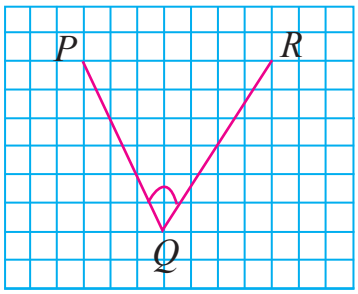
Right Angles

Any angle of magnitude 90° is called a "right angle". $K\hat{L}M$ is a right angle.



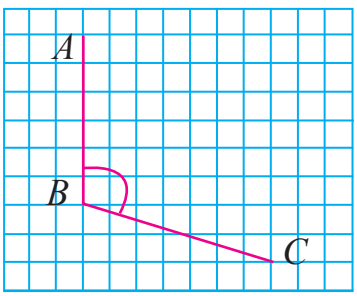
Acute Angles

Any angle of magnitude less than 90° is called an "acute angle". $P\hat{Q}R$ is an acute angle.



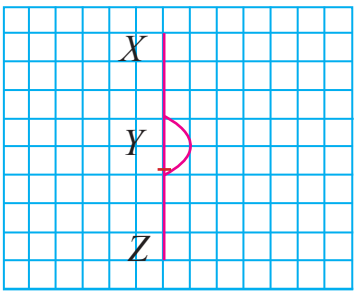
Obtuse Angles

Any angle of magnitude greater than 90° but less than 180° (that is an angle between 90°and 180°) is called an "obtuse angle". $A\hat{B}C$ is an obtuse angle.



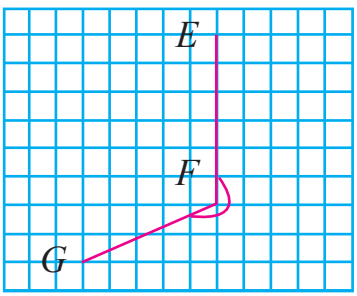
Straight Angles

Any angle of magnitude 180° is called a "straight angle". $X\hat{Y}Z$ is a straight angle.



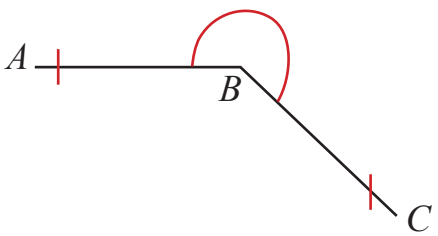
Reflex Angles

Any angle of magnitude between 180°and 360° is called a " reflex angle". $E\hat{F}G$ is a reflex angle.



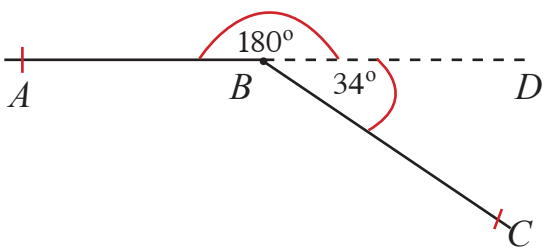
9.7 Measuring and Drawing Reflex angles

The figure shows the reflex angle $A\hat{B}C$.This angle cannot be measured directly using a protractor. So let us see how we can measure this reflex angle.



Method I :-

Let us use the ruler to extend AB and obtain the straight angle $A\hat{B}D$.
That is, $A\hat{B}D = 180^{o}$.

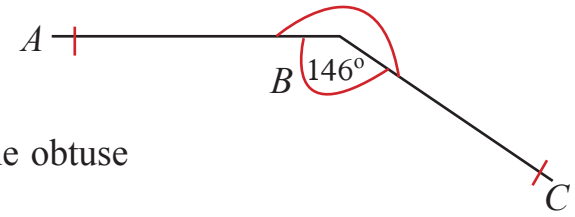


Now let us measure $D\hat{B}C$ using the protractor. We will then obtain $D\hat{B}C = 34^{o}$.

Since the reflex angle $A\hat{B}C = A\hat{B}D + D\hat{B}C$
the reflex angle $A\hat{B}C = 180^{o} + 34^{o} = 214^{o}$.

Method II :-

Measure the obtuse angle $A\hat{B}C$.
It is equal to146°.
Since the reflex angle $A\hat{B}C$ + the obtuse angle $A\hat{B}C = 360^{o}$



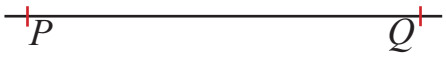
The reflex angle $A\hat{B}C = 360^{o} - 146^{o}$
$= 214^{o}$

Let us now see how to draw reflex angles.

Activity 4

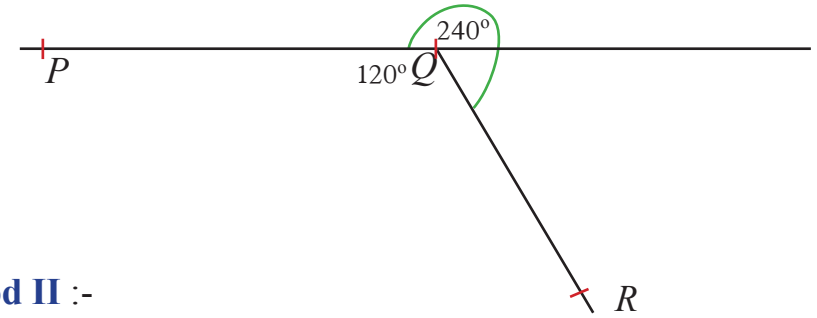
Draw the reflex angle $P\hat{Q}R = 240^{o}$ according to the following steps.

Step 1 - Draw the straight line segment PQ.



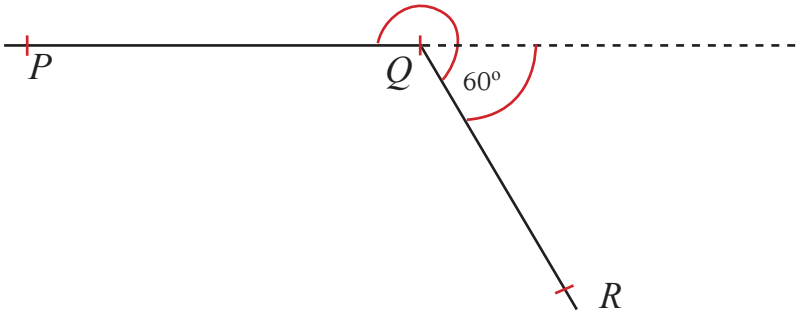
Step 2 - Calculate the magnitude of the obtuse angle $P\hat{Q}R$.
$P\hat{Q}R = 360^{o} - 240^{o} = 120^{o}$

Step 3 - Draw the angle $P\hat{Q}R = 120^{o}$. Now mark the reflex angle 240°.



Method II :-

Step 4 - By drawing an angle of 60° (That is, 240° – 180°) on the straight angle appropriately, we can obtain the reflex angle 240°.



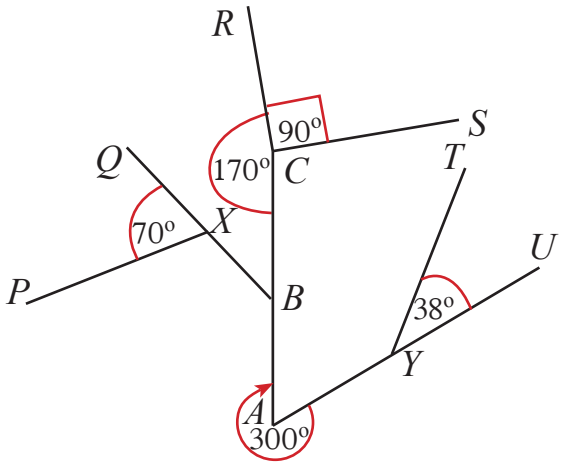
Exercise 9.5

(1) Copy the two groups (*a*) and (*b*) in your exercise book. Join each angle and its type with a straight line.

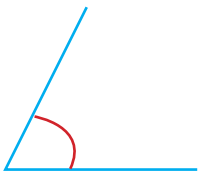
Group (*a*) (Magnitude of the angle)	Group (*b*) (Type of angle)
18°	Straight angle
135°	Right angle
180°	Acute angle
255°	Obtuse angle
90°	Reflex angle

(2) Using the information in the figure, write down the type of each of the angles given below.

(i) $P\hat{X}Q$ (ii) $B\hat{C}R$ (iii) $S\hat{C}R$ (iv) $T\hat{Y}U$ (v) $B\hat{A}Y$



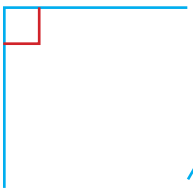
(3) Choose and write down the most appropriate magnitude for each of the angles below, from the values given in brackets.

(i) 

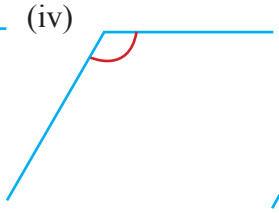
(25°, 65°, 10°)

(ii) 

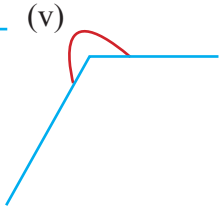
(1°, 80°, 15°)

(iii) 

(50°, 90°, 180°)

(iv) 

(360°, 120°, 180°)

(v) 

(185°, 240°, 350°)

(4) Draw the following reflex angles using the protractor.

(i) $A\hat{B}C = 300^{\circ}$ (ii) $P\hat{Q}R = 195^{\circ}$ (iii) $M\hat{N}O = 200^{\circ}$

(iv) $K\hat{L}M = 243^{\circ}$ (v) $X\hat{Y}Z = 310^{\circ}$

Summary

- The standard unit used to measure angles is degrees. One degree is written as 1°.
- Any angle of magnitude less than 90° is called an "acute angle".
- Any angle of magnitude 90° is called a "right angle".
- Any angle of magnitude greater than 90° but less than 180° (that is an angle between 90° and 180°) is called an "obtuse angle".
- Any angle of magnitude 180° is called a "straight angle".
- Any angle of magnitude between 180° and 360° is called a "reflex angle".