# 8 Directed Numbers

By studying this lesson you will be able to
- identify what directed numbers are,
- add integers using the number line, and
- add directed numbers without using the number line.

8.1 Identifying directed numbers

The figure given here represents an indicator that is used to measure the water level of a reservoir from which water is distributed to a certain city.

The usual water level of the reservoir has been marked as 0 (zero), and the indicator has been calibrated such that the gaps between the numbers above the 0 limit and below the 0 limit are equal.
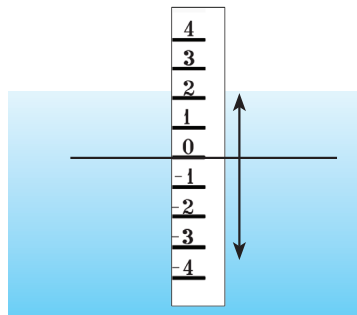
Thereby it can be observed whether the water level of the reservoir is above or below 0 (the usual level). Here, by calibrating the indicator in opposite directions, a correct perception of the water level of the reservoir is obtained.

Similarly, thermometers that are used to measure the temperature of the environment are calibrated in opposite directions from 0° C, to indicate temperatures that are greater than 0° C and temperatures that are less than 0° C.
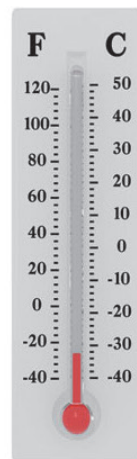
The thermometer in the figure has been calibrated with the values 10, 20, 30, … in one direction to indicate the temperatures that are greater than $0°$ C, and with the values $-10, -20, -30, …$ in the opposite direction to indicate the temperatures that are less than $0°$ C.

Let us now consider the number line given below.

The positive whole numbers marked to the right of the position indicating zero on the number line are defined as positive integers and the negative whole numbers marked to the left of the position indicating zero are defined as negative whole numbers.

$\{…, -3, -2, -1, 0, 1, 2, 3, …\}$ is the set consisting of all the integers.

Any positive number can be marked on the above number line to the right of the position indicating and any negative number can be marked to the left of the position indicating 0, taking into consideration the magnitude of the number.

All the numbers that are written with a positive or negative sign to indicate not only their magnitude but also one of two directions which are opposite to each other are defined as **directed numbers**.

Accordingly, numbers such as $+4, +\frac{3}{4}, +5.7, -10, -\frac{1}{3}$ and -3.2 are directed numbers. $+4$ is read as positive four and $-1\frac{3}{3}$ is read as negative one third.

Note
- When a sign is not written in front of a number, it is considered as a positive number.
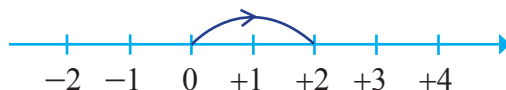
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8.2 Adding directed numbers which are integers by using the number line

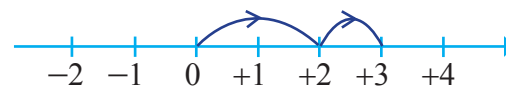Let us consider adding directed numbers which are positive integers by using the number line.

- ### The sum of two positive integers

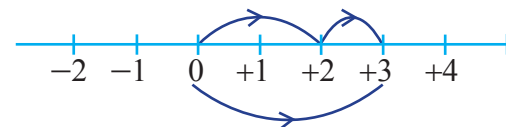Let us find the value of $(+2) + (+1)$ using the number line.

First, starting from 0, let us go two units towards the right along the number line.

Next, from this point, let us go one unit towards the right along the number line.
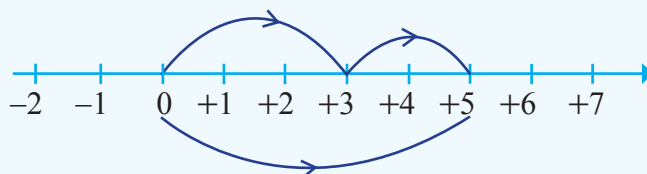
The directed number denoted by the position at which we finally stop is the answer.

$$(+2) + (+1) = (+3)$$

Example 1

Find the value of $(+3) + (+2)$ using the number line.

The final position is five units to the right of 0.

$\therefore (+3) + (+2) = (+5)$

Exercise 8.1

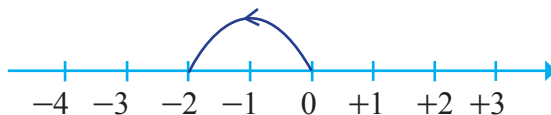Find each of the following sums using the number line.

(i) $(+2) + (+3)$ (ii) $(+3) + (+3)$ (iii) $(+4) + (+1)$ (iv) $(+5) + (+3)$
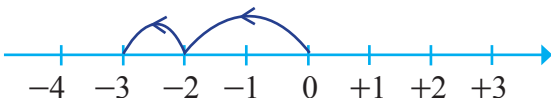
• The sum of two negative integers

Let us consider adding directed numbers which are negative integers by using the number line.

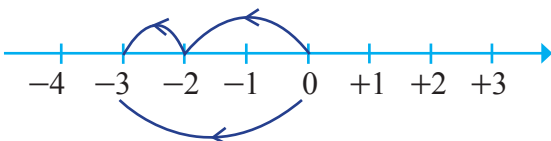Let us find the value of $(-2) + (-1)$ using the number line.

First, starting from 0, let us go two units towards the left along the number line.

Next, from this point, let us go one unit towards the left along the number line.

The directed number denoted by the position at which we finally stop is the answer.

$$(-2) + (-1) = (-3)$$

Example 1

Find the value of $(-3) + (-2)$ using the number line.

The final position is five units to the left of 0.
$\therefore (-3) + (-2) = (-5)$

Exercise 8.2

Find the value using the number line.

(i) $(-4) + (-1)$ (ii) $(-2) + (-2)$ (iii) $(-2) + (-3)$

(iv) $(-1) + (-3)$ (v) $(-3) + (-3)$ (vi) $(-4) + (-2)$

- ## The sum of a positive integer and a negative integer

Now let us consider adding a positive integer and a negative integer.

Let us find the value of $(+5) + (-2)$ using the number line.

First, starting from 0, let us go five units towards the right along the number line.

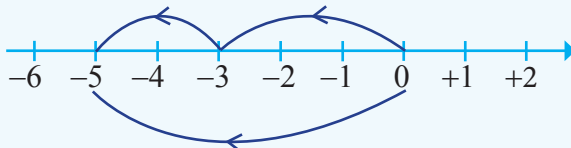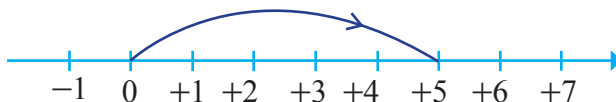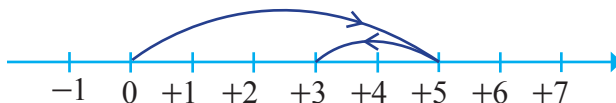Next, from this point, let us go two units towards the left along the number line.

The directed number denoted by the position at which we finally stop is the answer.
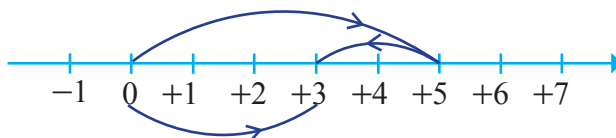
$(+5) + (-2) = (+3)$

Example 1

Find the value of $(-5) + (+2)$ using the number line.
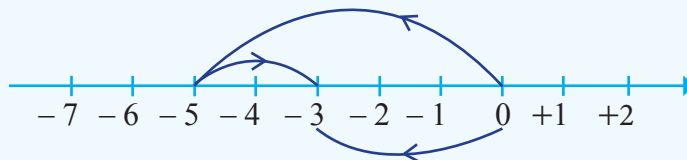
$(-5) + (+2) = (-3)$

Since the final position is three units to the left of 0, the number (-3) relevant to this position is the answer.

Exercise 8.3

Find the value using the number line.

(i) $(+3) + (-1)$ (ii) $(-4) + (+6)$ (iii) $(-7) + (+2)$

(iv) $(+2) + (-5)$ (v) $(+1) + (-1)$ (vi) $(-3) + (+3)$

8.3 Adding integers without using the number line

• Finding the sum of two integers

Let us consider the examples related to adding two positive integers that were studied in the previous section.

Using the number line we obtained previously that,

$(+2) + (+1) = (+3)$ and
$(+3) + (+2) = (+5)$.

$$(+2) + (+1) = (+3) \qquad\qquad (+3) + (+2) = (+5)$$
$$2 + 1 = 3 \qquad\qquad\qquad 3 + 2 = 5$$

☛ When adding two positive integers, add the two numbers without considering the signs.
☛ Place the positive sign in the final answer.

Let us now reconsider the examples related to adding two negative integers that were studied in the previous section.

Using the number line we obtained previously that

$(-2) + (-1) = (-3)$ and
$(-3) + (-2) = (-5)$.

Let us consider $(-2) + (-1) = (-3)$

☛ Without considering the signs of the two directed numbers, obtain their sum.
$$2 + 1 = 3$$
☛ Then write the answer with the negative sign. Therefore the answer is -3.

When adding two negative directed numbers, add the two numbers without considering the negative sign and then write the answer with the negative sign.

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Example 1

Simplify:

 (i) $(+4) + (+6)$ (ii) $(+11) + (+3)$ (iii) $(-5) + (-2)$ (iv) $(-4) + (-1)$

 (i) $(+4) + (+6) = (+10)$ (ii) $(+11) + (+3) = (+14)$

 (iii) $(-5) + (-2) = (-7)$ (iv) $(-4) + (-1) = (-5)$

Exercise 8.4

Simplify.

(i) $(+3) + (+8)$	(ii) $(-7) + (-3)$	(iii) $(+12) + (+4)$
(iv) $(-9) + (-16)$	(v) $(-20) + (-13)$	(vi) $(+17) + (+13)$
(vii) $(-11) + (-29)$	(viii) $(+2) + (+8)$	(ix) $(-3) + (-10)$

• **Finding the sum of a positive integer and a negative integer**

Using the number line we obtained previously that,
$(+5) + (-2) = (+3)$ and
$(-5) + (+2) = (-3)$.

We can find the sum of a positive integer and a negative integer as follows.

Let us consider $(-8) + (+5)$.
☛ Without considering the signs of the two directed numbers, obtain their difference. $8 - 5 = 3$
☛ From the two directed numbers (-8) and $(+5)$, the number which is further away from 0 on the number line is (-8). Its sign is negative.
☛ Therefore the answer is -3.

 $(-8) + (+5) = (-3)$

When adding two directed numbers of opposite signs (positive and negative), obtain their difference without considering the signs, and write the answer with the sign of the directed number which is further away from 0 on the number line.

Example 1	**Example 2**
Simplify (+8) + (−3) 8 − 3 = 5 From the two directed numbers (+8) and (−3), the number which is further away from 0 on the number line is (+8). Its sign is positive. (+8) + (−3) = (+5)	Simplify (+4) + (−10) 10 − 4 = 6 From the two directed numbers (+4) and (−10), the number which is further away from 0 on the number line is (−10). Its sign is negative. (+4) + (−10) = (−6)

Exercise 8.5

(1) Evaluate the following.

(i) (+7) + (−2)	(ii) (−10) + (+4)	(iii) (−3) + (+6)
(iv) (−5) + (+9)	(v) (−11) + (+4)	(vi) (−4) + 0
(vii) (+9) + (−8)	(viii) (+7) + (−15)	(ix) (+5) + (−6)
(x) (−7) + (+5)	(xi) (+8) + (−10)	(xii) (−9) + (+4)

8.4 Adding directed numbers

We have so far considered the addition of directed numbers which are integers. Now let us consider the addition of any two directed numbers.

The methods that were used above to add integers are used here too.

Example 1

Add the following directed numbers.

(i) $(+\frac{1}{2}) + (+\frac{1}{2})$

Without considering the signs of the two directed numbers, obtain their sum.

$$\frac{1}{2} + \frac{1}{2} = 1$$

Place the positive sign in the final answer.

$$(+\frac{1}{2}) + (+\frac{1}{2}) = +1$$

(ii) $(-\frac{2}{7}) + (-\frac{4}{7})$

Without considering the signs of the two directed numbers, obtain their sum.

$$\frac{2}{7} + \frac{4}{7} = \frac{6}{7}$$

Place the negative sign in the final answer.

$$(-\frac{2}{7}) + (-\frac{4}{7}) = (-\frac{6}{7})$$

(iii) $(+ 7.2) + (+ 1.3) = (+ 8.5)$

(iv) $(- 6.9) + (+ 2.5) = (- 4.4)$

Exercise 8.6

Evaluate the following.

(i) $(+\frac{3}{5}) + (+\frac{1}{5})$ (ii) $(-\frac{4}{7}) + (-\frac{1}{7})$ (iii) $(+\frac{2}{3}) + (+\frac{1}{3})$

(iv) $(- 2) + (-\frac{1}{2})$ (v) $(- 8.1) + (- 1.3)$ (vi) $(- 3.6) + (- 1.8)$

(vii) $(+ 4) + (- 2.5)$ (viii) $(-5) + (- 3.7)$ (ix) $(-\frac{4}{8}) + (-\frac{3}{8})$

(x) $(-2.6) + (+ 6.5) + (-4.3)$ (xi) $(+ 5.7) + (- 3.9) + (+ 1.4)$

Miscellaneous Exercise

(1) Fill in the blanks.

(i) $(+ 8) + (- 1) = (.....)$

(ii) $(+ 11) + (- 12) = (.....)$

(iii) $(- 4) + (- 11) = (.....)$

(iv) $(-\frac{7}{9}) + (-\frac{5}{9}) = (.....)$

(v) $(-\frac{8}{11}) + (-\frac{3}{11}) = (.....)$

(vi) $(+ 8.95) + (+ 2.97) = (.....)$

(vii) $(- 5.81) + (- 2.25) = (.....)$

(viii) $(- 6.57) + (+ 11.21) = (.....)$

(ix) $(-\frac{4}{13}) + (-\frac{7}{13}) = (.....)$

(x) $(+ 3.52) + (- 2.51) = (.....)$

(2) The ground floor of a building has been named Floor 0 and the floors above it have been named 1, 2, 3, … respectively, while the floors below it have been named −1, −2, −3, ... respectively.

  (i) If a person in Floor 7 climbs up a further 5 floors, which floor will he be in?

  (ii) If a person in Floor -1 descends a further 2 floors, which floor will he be in?

  (iii) If a person in Floor 8 descends 3 floors, which floor will he be in?

  (iv) If a person in Floor 2 descends 4 floors, which floor will he be in?

(3) The temperature at 6.00 a.m. in Moscow on a certain day was recorded as − 4.7° C, while the temperature at 4.00 p.m. on the same day was increased by 12^0 C. Find the temperature in Moscow at 4.00 p.m.

Summary

- All numbers that are written with a positive or negative sign to indicate not only their magnitude but also one of two directions which are opposite to each other are called directed numbers.
- When adding two directed numbers of the same sign, add the numbers without considering the sign, and then include the sign with the answer.
- When adding two directed numbers of opposite signs (positive and negative), obtain their difference without considering the signs, and write the answer with the sign of the directed number which is further away from 0 on the number line.

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