5 Indices

By studying this lesson you will be able to

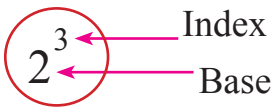
- write a number in index form, as a power having a prime number as the base,
- identify powers that have an algebraic symbol as the base,
- expand powers that have an algebraic symbol as the base and
- find the value of an algebraic expression by substituting positive integers for the unknowns.

Indices

Index notation is used to write a number which is multiplied repeatedly, in a concise way. Let us recall what has been learnt thus far about indices.

$2 \times 2 \times 2$ is written as 2^3 using indices.
That is, $2 \times 2 \times 2 = 2^3$.

In 2^3, 2 is defined as the base and 3 is defined as the index. 2^3 is read as "two to the power 3".



The value of 2^3 is 8. Therefore, the number 8 can be written as 2^3 in index notation.

When the index is a positive integer, it denotes how many times the number in the base is multiplied by itself.

Product	Number of times 3 is multiplied by itself	Index notation
3×3	2	3^2
$3 \times 3 \times 3$	3	3^3
$3 \times 3 \times 3 \times 3$	4	3^4
$3 \times 3 \times 3 \times 3 \times 3$	5	3^5
$3 \times 3 \times 3 \times 3 \times 3 \times 3$	6	3^6

You have learnt these facts in Grade 6. Do the following exercise to recall what you have learnt thus far about indices.

Review Exercise

(1) Expand each of the following as a product and find the value of the given expression.

(i) 3^2 (ii) 5^4 (iii) $2^2 \times 3$ (iv) $6^2 \times 5^2$

(2) Write down each of the following products using index notation.

(i) $4 \times 4 \times 4$ (ii) $7 \times 7 \times 7 \times 7$

(iii) $2 \times 2 \times 3 \times 3$ (iv) $3 \times 3 \times 5 \times 3 \times 5$

(3) Fill in the blanks in the following table.

Number	Index Notation	Base	Index	How the index notation is read
25	5^2	5	2	Five to the power two
343	.....	7	.....	.....
.....	.....	.....	.....	Six to the power three

(4) Write the number 16

(i) using index notation with base 2.

(ii) using index notation with base 4.

5.1 Expressing a number in index notation with a prime number as the base

Let us write 8 in index notation with a prime number as the base.

Let us write 8 as a product of its prime factors.

$$\begin{array}{r|l} 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array}$$

$8 = 2 \times 2 \times 2$

8 in index notation $= 2^3$

Now let us express the number 40 in index notation with prime numbers as the bases of the powers.

Let us write 40 as a product of prime numbers.

$40 = 2 \times 2 \times 2 \times 5$

$$\begin{array}{r|l} 2 & 40 \\ 2 & 20 \\ 2 & 10 \\ 5 & 5 \\ & 1 \end{array}$$

When this is written in index notation we obtain $40 = 2^3 \times 5$.

That is, 40 can be expressed as a product of powers with prime numbers as bases, in the form $40 = 2^3 \times 5$.

Do the following to express a number as a product of powers with prime numbers as bases.

- Start by dividing the number by the smallest prime number which divides it without remainder,
- Continue dividing the result by the prime numbers which divide it without remainder, in increasing order of the prime numbers, until the answer 1 is obtained.
- Write the number as a product of powers of these primes, where the index is the number of times division by that prime is done.

Example 1

Write down the number 36 as a product of powers with prime numbers as bases.

$$\begin{array}{r|l} 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$36 = 2 \times 2 \times 3 \times 3$

$36 = 2^2 \times 3^2$

Example 2

Write down the number 100 as a product of powers with prime numbers as bases.

$$\begin{array}{r|l} 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$100 = 2 \times 2 \times 5 \times 5$

$100 = 2^2 \times 5^2$

Exercise 5.1

(1) (i) Write 25 in index notation with 5 as the base.
(ii) Write 64 in index notation with 2 as the base.
(iii) Write 81 in index notation with 3 as the base.
(iv) Write 49 in index notation with 7 as the base.

(2) Write each of the following numbers as a product of powers with prime numbers as bases.
(i) 18 (ii) 24 (iii) 45 (iv) 63 (v) 72

5.2 Powers with an algebraic symbol as the base

We have learnt about powers with a number as the base. Let us now consider instances when the base is an algebraic symbol.

$2 \times 2 \times 2 = 2^3$

$5 \times 5 \times 5 = 5^3$

We can in a similar manner write $x \times x \times x = x^3$.

x^3 Index, Base

The base of x^3 is x and the index is 3.

Further,

$a \times a$ and $m \times m \times m \times m$ can be expressed as powers as $a \times a = a^2$ and $m \times m \times m \times m = m^4$ with an algebraic symbol as the base.

$2^1 = 2$. Accordingly, a can be written as $a = a^1$ in index notation.
The product of 2 and 3 is written as 2×3.
The product of a and b can be written as $a \times b$.
$a \times b$ can be expressed as ab or ba.
Accordingly $3 \times a \times b$ can be expressed as $3ab$.

Further, $m \times m \times m \times n \times n = m^3 \times n^2$.

$m^3 \times n^2$ which is also equal to $n^2 \times m^3$, can be expressed as m^3n^2 or as n^2m^3.

When two powers are connected with a multiplication sign, if the bases of both the powers are not numerical values, then it is not necessary to include the multiplication sign.

Example 1

Write down each of the following expressions using index notation.

(i) $p \times p \times p$ (ii) $x \times x \times y \times y \times y$

(iii) $2 \times 2 \times a \times a \times a$ (iv) $m \times 3 \times m \times 3 \times 3$

↳ (i) $p \times p \times p = p^3$ (ii) $x \times x \times y \times y \times y = x^2 \times y^3 = x^2y^3$

(iii) $2 \times 2 \times a \times a \times a = 2^2 \times a^3 = 2^2a^3$

(iv) $m \times 3 \times m \times 3 \times 3 = 3^3 \times m^2 = 3^3m^2$

Example 2

Expand and write each of the following expressions as a product.

(i) m^3 (ii) p^2q^3 (iii) 5^2x^3

↳ (i) $m^3 = m \times m \times m$ (ii) $p^2q^3 = p \times p \times q \times q \times q$

(iii) $5^2x^3 = 5 \times 5 \times x \times x \times x$

Exercise 5.2

(1) Write down each of the following expressions using index notation.

(i) $x \times x \times x \times x$ (ii) $a \times a \times a$ (iii) $m \times m \times m \times n \times n \times n$

(iv) $7 \times 7 \times 7 \times p \times p$ (v) $y \times y \times y \times y \times 7 \times 7 \times 7$

(2) Expand and write each of the following expressions as a product.

(i) a^2 (ii) $2p^2$ (iii) $2^3 m^2$ (iv) $3^2 x^3$ (v) $x^3 y^3$

5.3 Finding the value of a power by substitution

Let us consider expressions in index notation with bases which are unknowns. By substituting values for the unknown bases, the value of an expression in index notation can be found. In this lesson, only positive integers are substituted.

Let us find the value of the expression x^3 when $x = 2$.

Method I

By substituting the value 2 for x we obtain,

$x^3 = 2^3$
$= 2 \times 2 \times 2$
$= 8$

Method II

$x^3 = x \times x \times x$

By substituting the value 2 for x we obtain,

$x^3 = 2 \times 2 \times 2$
$x^3 = 8$

Example 1

Find the value of each of the following expressions when $x = 5$.

(i) x^3

Method I

$x^3 = 5^3$
$= 5 \times 5 \times 5$
$= 125$

Method II

$x^3 = x \times x \times x$
$= 5 \times 5 \times 5$
$= 125$

(ii) $3x$

$3x = 3 \times x$
$= 3 \times 5$
$= 15$

Example 2

Find the value of each of the following expressions when $a = 3$ and $b = 5$.

(i) $a^2 b$ (ii) $2a^3 b^2$

(i) $a^2 b$

$a^2 b = a \times a \times b$

Substituting $a = 3$ and $b = 5$ we obtain,

$a^2 b = 3 \times 3 \times 5$

$= 45$

(ii) $2a^3 b^2$

$2a^3 b^2 = 2 \times a \times a \times a \times b \times b$

Substituting $a = 3$ and $b = 5$ we obtain,

$2a^3 b^2 = 2 \times 3 \times 3 \times 3 \times 5 \times 5$

$= 1350$

Exercise 5.3

(1) Find the value of each of the following expressions by substituting $x = 3$.

(i) x^4 (ii) $3x^2$ (iii) $5x^3$

(2) Find the value of each of the following expressions by substituting $a = 3$.

(i) $2a^2$ (ii) 2^2a^2 (iii) $7a^2$

(3) Find the value of each of the following expressions by substituting $x = 1$ and $y = 7$.

(i) $x^2 y^3$ (ii) $2x^3 y$ (iii) $3x y^2$

(4) Find the value of each of the following expressions by substituting $a = 2$ and $b = 7$.

(i) $a^2 b$ (ii) ab^2 (iii) a^3b^2 (iv) $3a^2b^2$

Summary

- An expression of an unknown term multiplied repeatedly can be expressed as a power with the unknown term as the base and the number of times the term is multiplied as the index.

a to the power three → a^3 ← Index, Base

- When two powers are connected with a multiplication sign, if the bases of both the powers are not numerical values, then it is not necessary to include the multiplication sign.
- A value can be obtained for an expression in index notation with an unknown base, by substituting a number for the unknown term.