4 Factors and Multiples

(Part I)

By studying this lesson, you will be able to
- examine whether a whole number is divisible by 3, 4, 6 or 9.

4.1 Examining whether a number is divisible by 3, 4, 6 or 9

It is important to know the divisibility rules when solving problems related to factors and multiples.

If a certain whole number can be divided by another whole number without remainder, then the first number is said to be divisible by the second number. We then identify the second number as a factor of the first number.

$6 \div 2 = 3$ with remainder 0. Therefore, 2 is a factor of 6.

$6 \div 4 = 1$ with remainder 2. Therefore, 4 is not a factor of 6.

One way to find factors of numbers quickly is to use tests of divisibility.

The divisibility rules you learnt in grade 6 are as follows.

- If the ones place digit of a number is divisible by 2, then that number is divisible by 2.
- If the ones place digit of a number is either 0 or 5, then that number is divisible by 5.
- If the ones place digit of a number is 0, then that number is divisible by 10.

• The digital root

Now let us learn about the digital root of a number.

The digital root of a number is calculated by adding up all the digits of that number (and adding the digits of the sums if necessary), until a single digit from 1 to 9 is left. That single digit is defined as the **digital root** of the relevant number.

Let us see how the digital root of a number is found by considering the following example.
Let us find the digital root of 213. Let us add the digits of 213.
$2 + 1 + 3 = 6$
Then the digital root of 213 is 6.
The digital root of 242 = $2 + 4 + 2 = 8$

Let us find the digital root of 68.

$6 + 8 = 14$. Let us add the digits of 14. $1 + 4 = 5$.
$\therefore$ The digital root of 68 is 5.

It is possible to identify certain properties of a number by considering its digital root.

- **Examining whether a number is divisible by 9**

Let us do the following activity. Our goal is to identify a rule to examine whether a number is divisible by 9 or not.

Activity 1

Complete the following table and answer the given questions.

Number	Digital root	Remainder when divided by 9	Is the number divisible by 9?	Is 9 a factor?
45				
52				
134				
549				
1323				
1254				
5307				

(i) What are the digital roots of the numbers which are divisible by 9, in other words, the numbers of which 9 is a factor.
(ii) Using your answer to the previous part, suggest a method (other than division) to test whether a number is divisible by 9.

- If the digital root of a whole number is divisible by 9, then that number is divisible by 9. That is 9 is a factor of that number.

Examining whether a number is divisible by 3

Let us do the following activity. Our goal is to identify a rule to examine whether a number is divisible by 3 or not.

Activity 2

Complete the following table and answer the given questions.

Number	Digital root of the number	Is the digital root of the number divisible by 3?	Is the number divisible by 3?	Is 3 a factor?
15				
16				
24				
28				
210				
241				
372				
1269				

(i) What values do you get as the digital roots of the numbers which are divisible by 3, in other words, the numbers of which 3 is a factor?

(ii) Does 3 always divide the digital root of the numbers which are divisible by 3?

(iii) Is every number of which the digital root is indivisible by 3, indivisible by 3?

If the digital root of a whole number is divisible by 3, then that number is divisible by 3. That is 3 is a factor of that number.

Exercise 4.1

(1) Without dividing, select the numbers which are divisible by 9.

504, 652, 567, 856, 1143, 1351, 2719, 4536

(2) Without dividing, select and write down the numbers which are divisible by 3.

81, 102, 164, 189, 352, 372, 466, 756, 951, 1029

(3) 3 divides the number 65☐. Suggest two digits suitable for the empty space.

(4) Pencils were brought to be distributed among Nimal's friends on his birthday Party. The number of pencils was less than 150, but close to it. Nimal observed that each friend could be given 9 pencils. What is the maximum number of pencils that may have been brought?

(5) The following quantities of items were brought to make gift packs to be given to the winners of a competition.

131 exercise books — 130 pencils
128 platignum pens — 131 ballpoint pens

If each gift pack should contain 3 units of each item, write down the minimum extra amounts needed from each item.

• Examining whether a number is divisible by 6

You have learnt previously that, if the ones place digit of a number is zero or an even number, then that number is divisible by 2. You have also learnt how to determine whether a number is divisible by 3. Do the following activity to examine whether a number is divisible by 6.

Activity 3

Complete the following table and answer the given questions.

Number	Is the number divisible by 2 ?	Is the number divisible by 3 ?	Is the number divisible by 6 ?	Is 6 a factor?
95				
252				
506				
432				
552				
1236				

(i) Are all numbers which are divisible by 6, divisible by 2 also?
(ii) Are all numbers which are divisible by 6, divisible by 3 also?
(iii) Are all numbers which are divisible by 6, divisible by both 2 and 3?

(iv) Suggest a suitable method to identify the numbers which are divisible by 6, in other words, the numbers of which 6 is a factor.

If a number is divisible by both 2 and 3, then it is divisible by 6.

That is 6 is a factor of that number.

Examining whether a number is divisible by 4

In order to identify a rule to determine whether a number is divisible by 4, do the following activity.

Activity 4

Complete the following table and answer the given questions.

Number	Is the ones place digit divisible by 4?	Is the number formed by the last two digits divisible by 4?	Is the number divisible by 4?	Is 4 a factor?
36				
259				
244				
600				
658				
1272				
4828				

(i) Is the ones place digit of every number which is divisible by 4, divisible by 4?

(ii) Is the digital root of every number which is divisible by 4, divisible by 4?

(iii) Which of the above properties should be used to determine whether a number is divisible by 4?

If the last two digits of a whole number consisting of two or more digits is divisible by 4, then that number is divisible by 4. That is 4 is a factor of that number.

Exercise 4.2

(1) From the following, select and write down the numbers

(i) Which are divisible by 6.
(ii) Which are divisible by 4.

162, 187, 912, 966, 2118, 2123, 2472, 2541, 3024, 3308, 3332, 4800

(2) Write the following numbers in the appropriate column of the table given below. (A number may be written in both column (i) and column (iii).)

348, 496, 288, 414, 1024, 1272, 306, 258, 1008, 6700

(i)	(ii)	(iii)	(iv)
Numbers of which 4 is a factor	Reason for your selection	Numbers of which 6 is a factor	Reason for your selection

(3) The number 62☐6 is divisible by both 4 and 6. Find the suitable digit for the empty space.

(4) A drill team arranges themselves in the following manner. On one occasion they form lines consisting of 3 members each and on another occasion lines consisting of 4 members each. They also make circles of 9 members each. If the drill team must have more than 250 members, use the divisibility rules to find the minimum number of members that could be in the team.

(5) Determine whether 126 is divisible by 2, 3, 4, 5, 6, 9 and 10.

Summary

Number	Divisibility Rule
2	If 2 divides the ones place digit of a whole number, then that number is divisible by 2.
3	If the digital root of a whole number is divisible by 3, then that number is divisible by 3.
4	If the last two digits of a whole number consisting of two or more digits is divisible by 4, then that whole number is divisible by 4.
5	If the ones place digit of a whole number is either 0 or 5, then that number is divisible by 5.
6	If a whole number is divisible by both 2 and 3, then it is divisible by 6.
9	If the digital root of a whole number is 9, then that number is divisible by 9.
10	If the ones place digit of a whole number is 0, then that number is divisible by 10.

4

Factors and Multiples
(Part II)

By studying this lesson, you will be able to
- find the factors of a whole number,
- write the multiples of a whole number,
- write the prime factors of a whole number,
- find the highest common factor (HCF) of a whole number and
- find the least common multiple (LCM) of a whole number.

4.2 Factors and multiples of a whole number

You learnt in grade six, how to find the factors and multiples of a whole number. Let us recall what you learnt.

Let us find the factors of 36.

Let us factorize 36 by expressing it as a product of two whole numbers.

$$36 = 1 \times 36$$
$$36 = 2 \times 18$$
$$36 = 3 \times 12$$
$$36 = 4 \times 9$$
$$36 = 6 \times 6$$

When a whole number is written as a product of two whole numbers, those two numbers are known as **factors** of the original number.

Therefore, the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

Let us factorize 126, using the method of division.

$$\begin{array}{r|l} 2 & \underline{126} \\ & \ \ 63 \end{array}$$

Since the number 126, can be divided by 2 without remainder, 2 is a factor of 126.
Since, $2 \times 63 = 126$, we obtain that 63 is also a factor of 126.

$$\begin{array}{r|l} 3 & \underline{126} \\ & \ \ 42 \end{array} \quad \begin{array}{r|l} 6 & \underline{126} \\ & \ \ 21 \end{array} \quad \begin{array}{r|l} 7 & \underline{126} \\ & \ \ 18 \end{array} \quad \begin{array}{r|l} 9 & \underline{126} \\ & \ \ 14 \end{array} \quad \begin{array}{r|l} 14 & \underline{126} \\ & \ \ \ \ 9 \end{array}$$

$2 \times 63 = 126$ $\quad$ $3 \times 42 = 126$ $\quad$ $6 \times 21 = 126$

$7 \times 18 = 126$ $\quad$ $9 \times 14 = 126$ $\quad$ $14 \times 9 = 126$

Therefore, the factors of 126 are 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63 and 126.

Note:
The divisibility rules can be used to determine whether a given number is divisible by another number or not.

Now let us consider how multiples of a whole number are found.

Let us compute the multiples of 13.
This can be done by multiplying 13 by whole numbers.

$13 \times 1 = 13$ $13 \times 2 = 26$ $13 \times 3 = 39$ $13 \times 4 = 52$

13, 26, 39 and 52 are a few examples of multiples of 13. Note that 13 is a factor of each of them. Therefore, any number of which 13 is a factor, is a multiple of 13.

Exercise 4.3

(1) Factorize.
(i) 150 (ii) 204 (iii) 165 (iv) 284

(2) Write down the ten factors of 770 below 100.

(3) (i) Write five multiples of 36.
(ii) Write five multiples of 112.
(iii) Write five multiples of 53 below 500.

(4) 180 chairs in an examination hall have to be arranged such that each row has an equal number of chairs. If the minimum number of chairs that should be in a row is 10 and the maximum that could be in a row is 15, find how many possible ways there are to arrange the chairs.

4.3 Prime factors of a whole number

You have already learnt that whole numbers greater than one with exactly two distinct factors are called prime numbers.

Let us recall the prime numbers below 20.

They are 2, 3, 5, 7, 11, 13, 17 and 19.

Let us identify the prime factors of 36. We learnt above that the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

There are only two prime numbers among them, namely, 2 and 3. These are the prime factors of 36.

Let us find the prime factors of 60.
The factors of 60 are 1, 2, 3, 4, 5, 6, 12, 15, 20, 30 and 60.
The prime factors among them are 2, 3 and 5.

The prime numbers among the factors of a number are its prime factors.

Any whole number which is not a prime number can be expressed as a product of its prime factors.

A method of finding the prime factors of a whole number using the method of division and writing the number as a product of its prime factors is described below.

Let us find the prime factors of 84 and write it as a product of its prime factors.

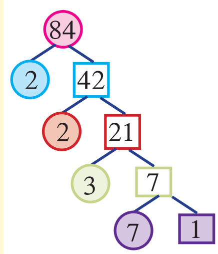
- Here 84 has been divided by 2, the smallest prime number.
- Division by 2 is continued, until a number which is not divisible by 2 is obtained.
- When this result is divided by the next smallest prime, which is 3, the result 7 is obtained. When this is divided by the prime number 7, the answer obtained is 1.
- In this manner, we continue dividing by prime numbers until the answer 1 is obtained.

Accordingly, the prime factors of 84 are 2, 3 and 7, which are the numbers by which 84 was divided.

Now, to write 84 as a product of its prime factors, write it as a product of the prime numbers by which it was divided.

$84 = 2 \times 2 \times 3 \times 7$

2	84
2	42
3	21
7	7
	1

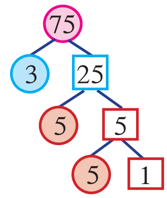


Let us write 75 as a product of its prime factors. Let us divide 75 by prime numbers.

- Since 75 is not divisible by 2, we divide it by 3, the next smallest prime number.
- The result 25 is not divisible by 3.
- When 25 is divided twice by 5 which is the next smallest prime number, the result is 1.

Accordingly, when 75 is written as a product of its prime factors we obtain $75 = 3 \times 5 \times 5$.

3	75
5	25
5	5
	1



- When finding the prime factors of a whole number, the number is divided by the prime numbers which divide it without remainder, starting from the smallest such prime number, until the answer 1 is obtained.
- The prime numbers which divide a number without remainder are its prime factors.
- To write a number as a product of its prime factors, write it as a product of the prime numbers by which it was divided.

Example 1

Write 63 as a product of its prime factors.

3	63
3	21
7	7
	1

Since 63 is not divisible by 2, division is started with 3. The result 21 is again divided by 3. Then we obtain 7 which is not divisible by 3. We divide this by 7 to obtain 1.

Therefore, 63 written as a product of its prime factors is
$63 = 3 \times 3 \times 7$.

Exercise 4.4

(1) Find the prime factors of each of the following numbers.

(i) 81 (ii) 84 (iii) 96

(2) Express each of the following numbers as a product of its prime factors.

(i) 12 (ii) 15 (iii) 16 (iv) 18 (v) 20
(vi) 28 (vii) 59 (viii) 65 (ix) 77 (x) 91

4.4 Finding the factors of a number by considering its prime factors

Suppose we need to find the factors of 72.
Let us start by writing 72 as a product of its prime factors.

2	72
2	36
2	18
3	9
3	3
	1

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 1 \times 72$

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2 \times 36$

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 4 \times 18$

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 8 \times 9$

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 24 \times 3$

The factors of a whole number (which are not its prime factors or 1) can be obtained by taking products of 2 or more of its prime factors.

2, 36, 4, 18, 8, 9, 24 and 3 are eight factors of 72. 1 and 72 are also factors of 72.

1, 2, 3, 4, 8, 9, 18, 24, 36 and 72 are ten factors of 72.

Exercise 4.5

(1) Find six factors of each of the following numbers by considering their prime factors.

(i) 20 (ii) 42 (iii) 70 (iv) 84 (v) 66 (vi) 99

4.5 Highest Common Factor (HCF) (Greatest Common Divisor (GCD))

Let us now consider what the highest common factor (HCF) of several numbers is and how it is found.

Let us find the highest common factor of the numbers 6, 12 and 18.

- Write down the factors of these numbers as follows.

Factors of 6: 1, 2, 3 and 6
Factors of 12: 1, 2, 3, 4, 6 and 12
Factors of 18: 1, 2, 3, 6, 9 and 18

- Circle and write the factors common to all three numbers.

  The factors which are common to 6, 12 and 18 are, 1, 2, 3 and 6.
- The largest number among the common factors is the **highest common factor** of these numbers.

We observe that the largest or the greatest of these common factors is 6. Therefore, 6 is the **highest common factor** of 6, 12 and 18.

Thus, the highest common factor of 6, 12 and 18 is 6, which is the largest number by which these three numbers are divisible.

- Given two or more numbers, the largest number which is a factor of all of them is known as their highest common factor (HCF).
- Accordingly, the highest common factor is the largest number by which all the given numbers are divisible.
- If 1 is the only common divisor of several numbers, then the highest common factor of these numbers is 1.

• Finding the highest common factor by writing each number as a product of its prime factors

Let us find the highest common factor of 6, 12 and 18.

- Let us write each number as a product of its prime factors.

$6 = 2 \times 3$
$12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$

2	6
3	3
	1

2	12
2	6
3	3
	1

2	18
3	9
3	3
	1

$6 = 2 \times 3$ $\quad 12 = 2 \times 2 \times 3$ $\quad 18 = 2 \times 3 \times 3$

- The highest common factor is obtained by taking the product of the prime factors which are common to all three numbers.

2 and 3 are the common prime factors of 6, 12 and 18.
Thus, the HCF of 6, 12 and 18 is $2 \times 3 = 6$.

• Finding the highest common factor by the method of division

Let us find the highest common factor of 6, 12 and 18.

2	6,	12,	18
3	3,	6,	9
	1,	2,	3

- Write the numbers as shown.
- Since all these numbers are divisible by 2, divide each of them by 2 individually.
- The result is 3, 6 and 9. Since 3, 6 and 9 are divisible by 3, the next smallest prime number, divide them by 3 and write the result below the respective numbers.
- The result is 1, 2 and 3. Since there isn't a prime number which divides all of 1, 2 and 3 without remainder, the division is stopped here.
- The HCF is obtained by multiplying the numbers by which division was done.

Thus the HCF of 6, 12 and 18 is $2 \times 3 = 6$.

When using the method of division to find the HCF,
- keep dividing all the numbers by the prime numbers which divide all the numbers without remainder.
- then multiply all the divisors and obtain the HCF of the given numbers.

The HCF of any set of prime numbers is 1.

Example 1

Find the highest common factor of 72 and 108.

Method I

The factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36 and 72.
The factors of 108 are 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54 and 108.

When the factors common to both these numbers are selected we obtain 1, 2, 3, 4, 6, 9, 12, 18 and 36.

Since the greatest of these common factors is 36, the highest common factor of 72 and 108 is 36.

Method II

Let us write 72 and 108 as products of their prime factors.

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$72 = 2 \times 2 \times 2 \times 3 \times 3$
$108 = 2 \times 2 \times 3 \times 3 \times 3$

The prime factors which are common to both the numbers 72 and 108 are 2, 2, 3 and 3.

Accordingly, the HCF of 72 and 108 $\} = 2 \times 2 \times 3 \times 3 = 36$

Method III

Find the HCF of 72 and 108.

$$\begin{array}{r|l} 2 & 72,\ 108 \\ \hline 2 & 36,\ 54 \\ \hline 3 & 18,\ 27 \\ \hline 3 & 6,\ 9 \\ \hline & 2,\ 3 \end{array}$$

Since there isn't another prime number which divides both 2 and 3 without remainder, stop the division here.

The HCF of 72 and 108 $\} = 2 \times 2 \times 3 \times 3$
$= 36$

The HCF 36 of 72 and 108 can also be described as the largest number that divides them both without remainder.

Example 2

(1) Three types of items were brought in the following quantities to be offered at an almsgiving.

30 cakes of soap
24 tubes of toothpaste
18 bottles of ointment

Parcels were made such that each contained all three items. Every parcel had the same number of items of a particular type. What is the maximum number of parcels that can be made accordingly? Write down the quantity of each item in a single parcel.

Every parcel should contain the same number of items of a particular type. To find the maximum number of parcels that can be made, we need to find the largest number which divides 30, 24 and 18 without remainder.

Therefore, let us find the HCF of 30, 24 and 18.

$$30 = 2 \times 3 \times 5$$
$$24 = 2 \times 2 \times 2 \times 3$$
$$18 = 2 \times 3 \times 3$$

HCF of 30, 24 and 18 = $2 \times 3 = 6$.

The maximum number of parcels that can be made = 6

The number of cakes of soap in a parcel = $30 \div 6 = 5$

The number of tubes of toothpaste in a parcel = $24 \div 6 = 4$

The number of bottles of ointment in a parcel = $18 \div 6 = 3$

Exercise 4.6

(1) Fill in the blanks to obtain the HCF by writing down all factors of the given numbers.

(i) Factors of 8 are ...., ..., ... and .....
Factors of 12 are ...., ..., ..., ..., ..., and .....
Factors common to 8 and 12 are ...., ..., and .....
∴ The HCF of 8 and 12 is .... .

(ii) 54 written as a product of its prime factors = 2 × ..... × 3 × .....
90 written as a product of its prime factors = ... × 3 × ..... × 5.
72 written as a product of its prime factors = 2 × 2 × ..... × ..... × .....
∴ The HCF of 54, 90 and 72 = ... × ..... × .....
= .....

(2) Find the HCF of each pair of numbers by writing down all their factors.

(i) 12, 15 (ii) 24, 30 (iii) 60, 72
(iv) 4, 5 (v) 72, 96 (vi) 54, 35

(3) Find the HCF of each pair of numbers by writing each number as a product of its prime factors.

(i) 24, 36 (ii) 45, 54 (iii) 32,48 (iv) 48, 72 (v) 18, 36

(4) Find the HCF by any method you like.

(i) 18, 12, 15 (ii) 12, 18, 24 (iii) 24, 32, 48 (iv) 18, 27, 36

(5) A basket contains 96 apples and another basket contains 60 oranges. If these fruits are to be packed into bags such that there is an equal number of apples in every bag and an equal number of oranges too in every bag and no fruits remain after they are packed into the bags, what is the maximum number of such bags that can be prepared?

4.6 Least Common Multiple (LCM)

Now let us consider what is meant by the least common multiple of several numbers and how it is found.

As an example, let us find the least common multiple of the numbers 2, 3 and 4.

- List the multiples of the given numbers.

Several multiples of the numbers 2, 3 and 4 are given in the following table.

Multiples of 2	2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26
Multiples of 3	3, 6, 9, 12, 15, 18, 21, 24
Multiples of 4	4, 8, 12, 16, 20, 24, 28

- Circle and write down the common multiples.

You will observe that the common multiples of the three numbers listed here are 12 and 24.

Further, if we continue to write the common multiples of 2, 3 and 4, we will obtain 12, 24, 36, 48, 60 etc

- The smallest of the common multiples of several numbers is called the **least common multiple** (LCM) of these numbers.

The smallest or the least of the common multiples 12, 24, 36, 48, 60, … of the numbers 2, 3 and 4 is 12.
Therefore, 12 is the least common multiple of 2, 3 and 4.

In other words, the smallest number which is divisible by 2, 3 and 4 is the least common multiple of 2, 3 and 4.

The least common multiple of several numbers is the smallest positive number which is divisible by all these numbers.

Note

- The HCF of several numbers is equal to or smaller than the smallest of these numbers
- The LCM of several numbers is equal to or larger than the largest of these numbers.
- The HCF of any two numbers is smaller than the LCM of the two numbers.

• Finding the LCM of several numbers by considering their prime factors

Let us see how the LCM of several numbers is found by considering their prime factors.

Let us find the LCM of 4, 12 and 18.

- Let us write each number as a product of its prime factors.

$$4 = 2 \times 2 = 2^2$$
$$12 = 2 \times 2 \times 3 = 2^2 \times 3^1$$
$$18 = 2 \times 3 \times 3 = 2^1 \times 3^2$$

- Let us select the greatest power of each prime factor.

There are two distinct prime factors, namely, 2 and 3. When the factors of all three numbers are considered,

the power of 2 with the largest index = 2^2
the power of 3 with the largest index = 3^2.

- The LCM of the given numbers is the product of these greatest powers.
  Therefore, the LCM of 4, 12 and 18 $= 2^2 \times 3^2$
  $= 2 \times 2 \times 3 \times 3$
  $= 36$

• Finding the LCM by the method of division

Let us find the LCM of 4, 12 and 18.

2	4,	12,	18
2	2,	6,	9
3	1,	3,	9
	1,	1,	3

- Write these numbers as shown.
- Since all these numbers are divisible by 2, divide each of them by 2 individually.
- We get 2, 6 and 9 as the result. No prime number divides all of them without remainder. However, 2 divides both 2 and 6 without remainder. Divide 2 and 6 by 2, and write the results below the respective numbers. Write 9 below 9.
- Since 3 and 9 are divisible by 3, the next smallest prime number, divide them by 3 and write the results below the respective numbers. Now observe that we cannot find at least two numbers which are divisible by the same number. Therefore, the division is stopped here.
- Multiply all divisors and all numbers left in the last row. The product gives the LCM of the given numbers.
  Accordingly, the LCM of 4, 12 and 18 $= 2 \times 2 \times 3 \times 1 \times 1 \times 3 = 36$

Note
When using the method of division to find the LCM, keep dividing if there remain at least two numbers, divisible by another and obtain the LCM of the given numbers as above.

Let us find the LCM of 4, 3 and 5.

Here, we do not have at least two numbers which are divisible by a common number which is greater than 1.

Therefore, the LCM of 4,3 and 5 $= 4 \times 3 \times 5$
$= 60$

Example 1

Find the LCM of 8, 6 and 16.

Method I

Let us write each number as a product of its prime factors as follows.

$8 \quad = 2 \times 2 \times 2 \qquad = 2^3$

$6 \quad = 2 \times 3 \qquad = 2^1 \times 3^1$

$16 \quad = 2 \times 2 \times 2 \times 2 \qquad = 2^4$

The distinct prime factors of these three numbers are 2 and 3. Here, the maximum number of times that 2 appears is four. The maximum number of times that 3 appears is one.

Therefore, the LCM of 8, 6 and 16 $\Big\} = 2^4 \times 3$
$= 2 \times 2 \times 2 \times 2 \times 3$
$= 48$

Method II

2	8,	6,	16
2	4,	3,	8
2	2,	3,	4
	1,	3,	2

Since we cannot find another prime number by which at least two of the three numbers 1, 3, 2 are divisible, we stop the division here.

The LCM of 8, 6 and 16
$= 2 \times 2 \times 2 \times 1 \times 3 \times 2 = 48$

Example 2

(1) Two bells ring at intervals of 6 minutes and 8 minutes respectively. If they both ring together at 8.00 a.m., at what time will they ring together again?

↳ To find when both bells ring together again, it is necessary to find at what time intervals the bells ring together.

The first bell rings once every 6 minutes. 6, 12, 18, 24, …
The second bell rings once every 8 minutes. 8, 16, 24, …
Therefore, the two bells ring together again after 24 minutes.

The answer could be obtained by finding the LCM too.
Since the bells ring together at intervals of the LCM of 6 minutes and 8 minutes, to find out when the two bells ring together again, the LCM of these two numbers needs to be found.

∴ let us find the LCM of 6 and 8.

$$\begin{array}{r|l} 2 & 6,\ 8 \\ \hline & 3,\ 4 \end{array}$$

The LCM of 6 and 8 = $2\times3\times4 = 24$.
Therefore, the two bells ring together again after 24 minutes.
The bells ring together initially at 8.00 a.m.
Therefore, the bells ring together again at 8.24 a.m.

Exercise 4.7

(1) Find the LCM of each of the following triples of numbers.

(i) 18, 24, 36 (ii) 8, 14, 28 (iii) 20, 30, 40
(iv) 9, 12, 27 (v) 2, 3, 5 (vi) 36, 54, 24

(2) At a military function, three cannons are fired at intervals of 12 seconds. 16 seconds and 18 seconds respectively. If the three cannons are fired together initially, after how many seconds will they all be fired together again?

Miscellaneous Exercise

(1) Without dividing, determine whether the number 35 343 is divisible by 3, 4, 6 and 9.

(2) Fill in the blanks.

(i) The HCF of 2 and 3 is .... .
(ii) The LCM of 4 and 12 is .... .
(iii) The HCF of two prime numbers is .... .
(iv) The LCM of 2, 3 and 5 is .... .

(3) Find the HCF and LCM of 12, 42 and 75.

(4) It is proposed to distribute books among 45 students in a class, such that each student receives no less than 5 books and no more than 10 books. Find all possible values that the total number of books that need to be bought can take, if all the students are to receive the same number of books and no books should be left over.

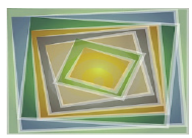
Summary

- Prime numbers among the factors of a number are called the prime factors of that number.
- Given two or more numbers, the largest of their common factors is called the highest common factor (HCF) of these numbers. Thus, the largest number which divides a list of numbers is their HCF.
- Given two or more numbers, the smallest of their common multiples is called the least common multiple (LCM) of these numbers. Thus, the smallest positive number which is divisible by a list of numbers is their LCM.

Ponder

(1) The length and breadth of a rectangular piece of cloth are 16 cm and 12 cm respectively. If this piece is to be cut without any waste into equal sized square pieces, what is the maximum possible side length of such a square piece?

(2) The length and the breadth of a floor tile are 16 cm and 12 cm respectively. What is the side length of the smallest square shaped floor area that can be tiled using such tiles, if no tile is to be cut?



(3) The circumference of the front and a back wheel of a tricycle are 96 cm and 84 cm respectively. What is the minimum distance the tricycle must move, for the front wheel and back wheels to complete full revolutions at the same instant?

(4) What is the smallest number which is greater than 19 that has a remainder of 19, when divided by 24, 60 and 36?