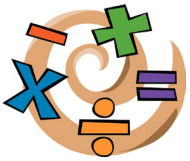
3 Operations on whole numbers

By studying this lesson you will be able to

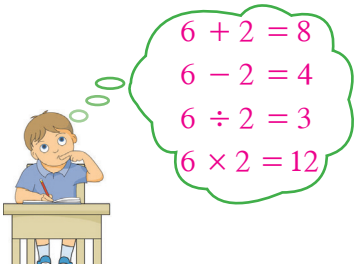
- identify the convention used in simplifying numerical expressions, and
- simplify numerical expressions consisting of whole numbers.

3.1 Mathematical operations on two whole numbers



Addition, multiplication, subtraction and division are symbolized by +, ×, − and ÷ respectively.

You have already learnt how to add and multiply two whole numbers.



Further, you know how to subtract one whole number from another, and how to divide one whole number by another.

Here each mathematical operation was performed only once.

3.2 The order in which mathematical operations in a numerical expression are performed

Consider the expression $3 + 7 \times 5$.

This is a numerical expression with three whole numbers and two operations.

Here + and × are defined as the operations of this expression.
The order in which the operations appear is + first and then ×.

If we consider the expression $15 \div 3 - 2$, the order in which the operations appear is ÷ first and then −.

Example 1

Write down the operations of the expression $12 \times 2 - 5 \times 3$ in the order in which they appear.

The order in which the operations appear is $\times$, $-$ and $\times$.

Exercise 3.1

(1) For each of the following numerical expressions, write down the mathematical operations in the order in which they appear.

(i) $5 + 3 + 2$ (ii) $6 \times 3 - 6$ (iii) $10 - 8 \div 2 \times 3$

(iv) $11 \times 2 + 5 - 2$ (v) $24 \div 6 + 6 \div 3$

3.3 Simplifying numerical expressions

• Simplifying expressions involving only addition

Let us simplify the expression $8 + 7 + 2$ in two different ways.

Let us add 8 and 7 first, and then add 2 to the result. This yields the answer 17.

$8 + 7 + 2 = 15 + 2 = 17$

Adding 7 and 2 first, followed by adding 8 to the result also yields the answer 17.

$8 + 7 + 2 = 8 + 9 = 17$

• Simplifying expressions involving only multiplication

Let us simplify the expression $5 \times 2 \times 3$ in two different ways.

Multiplying 5 by 2 first, and then multiplying the result by 3 yields the answer 30. $5 \times 2 \times 3 = 10 \times 3 = 30$

Multiplying 2 by 3 first, and then multiplying the result by 5 also yields the answer 30.

$5 \times 2 \times 3 = 5 \times 6 = 30$

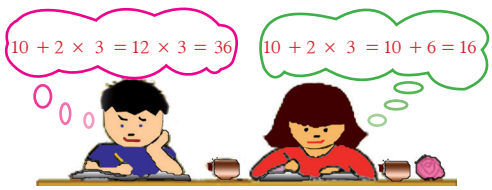
Thus, if either addition or multiplication is the only operation in a numerical expression, then irrespective of the order in which the operations are performed, the result obtained is the same.

Exercise 3.2

(1) Simplify each of the following expressions.

(i) $12 + 5 + 8$ (ii) $5 \times 8 \times 3$ (iii) $7 + 3 + 2 + 6$ (iv) $2 \times 5 \times 4 \times 3$

3.4 Further simplification of numerical expressions



Let us simplify the expression $10 + 2 \times 3$. Let us compare the answers we obtain when we simplify $10 + 2 \times 3$ by performing the operations in two different orders.

Let us first add 10 to 2 and then multiply the answer by 3.
$10 + 2 \times 3 = 12 \times 3 = 36$.

Now let us multiply 2 by 3, and then add 10 to it.
$10 + 2 \times 3 = 10 + 6 = 16$

Therefore, it is clear that when numbers simplify such numerical expressions which involve more than two terms and several operations, we may end up with different answers, depending on the order in which we perform the operations.

This emphasises the need for a convention when simplifying expressions involving two or more operations.

Let us consider below the convention used when simplifying such expressions.

- First perform all divisions (÷) and multiplications (×), working from left to right.
- Then perform all additions (+) and subtractions (–), working from left to right.

Only the operations of addition and multiplication appear in the expression $10 + 2 \times 3$. According to the above convention, multiplication should be performed first.

$10 + 2 \times 3 = 10 + 6 = 16$

Also, if only subtraction (–) and addition (+), or only division (÷) and multiplication (×) appear in a numerical expression, simplification is done from left to right in the order that the operations appear.

➤ Simplifying expressions involving only addition and subtraction

Let us simplify the expression $10 - 7 + 2$.

Here the order in which the operations appear from left to right is – first and then +.

When simplifying 10 – 7 + 2, first 7 is subtracted from 10 and then 2 is added to the result.

$\therefore\ 10 - 7 + 2 = 3 + 2 = 5$

Another example is: $6 + 7 - 2 = 13 - 2 = 11$

➤ Simplifying expressions involving only multiplication and division

Let us deal with the expression $36 \div 6 \times 3$ in a similar way.
Here the order in which the operations appear from left to right is ÷ first and then ×.

Let us first divide 36 by 6, and then multiply the answer by 3.
We then obtain $36 \div 6 \times 3 = 6 \times 3 = 18$

Another example is: $36 \times 6 \div 3 = 216 \div 3 = 72$

➤ Simplifying expressions in which only subtraction (–) or division (÷) appears several times.

When simplifying expressions involving only subtraction (–) or division (÷), the order in which the operations are performed is from left to right.

Consider the expression 10 – 3 – 2, where the operation of subtraction is applied twice. When the expression 36 ÷ 6 ÷ 3 is considered, division is applied twice.

Let us simplify these expressions.

Now let us subtract 3 from 10, and then subtract 2 from the result.
10 – 3 – 2 = 7 – 2 = 5.
Let us divide 36 by 6, and then divide the answer by 3.We then obtain,
36 ÷ 6 ÷ 3 = 6 ÷ 3 = 2.

Example 1

Simplify 7 – 4 + 5.

7 – 4 + 5 = 3 + 5
= 8

Example 2

Simplify 80 ÷ 10 × 5.

80 ÷ 10 × 5 = 8 × 5
= 40

Example 3

Simplify 4 × 6 ÷ 3.

4 × 6 ÷ 3 = 24 ÷ 3
= 8

Example 4

Simplify 25 + 10 – 7.

25 + 10 – 7 = 35 – 7
= 28

Example 5

Simplify 28 ÷ 2 – 3.

28 ÷ 2 – 3 = 14 – 3
= 11

Example 6

Simplify 50 – 10 × 3.

50 – 10 × 3 = 50 – 30
= 20

Example 7

Simplify 18 × 5 – 62.

18 × 5 – 62 = 90 – 62
= 28

Example 8

Simplify 50 –10 ÷ 2.

50 – 10 ÷ 2 = 50 – 5
= 45

Example 9

Simplify 5 + 6 ÷ 3 + 2.

5 + 6 ÷ 3 + 2 = 5 + 2 + 2
= 9

Example 10

Simplify 2 × 12 ÷ 3 × 5.

2 × 12 ÷ 3 × 5 = 24 ÷ 3 × 5
= 8 × 5 = 40

Exercise 3.3

(1) Place a ✓ next to the correct statements and a × next to the incorrect statements.

(i) $8 - 5 + 2 = 1$ (ii) $12 \times 3 - 11 = 25$

(iii) $7 + 18 \div 6 = 10$ (iv) $5 \times 6 \div 3 + 7 = 3$

(2) Simplify the following expressions.

(i) $10 \times 4 + 17$ (ii) $8 \times 3 + 5$ (iii) $14 \div 7 \times 5$

(iv) $448 + 12 \div 3$ (v) $7 \times 200 + 108$ (vi) $8 \times 9 - 61$

(vii) $100 - 7 \times 8$ (viii) $195 - 12 \times 10 \div 5$ (ix) $7 + 5 \times 37 + 278$

• Simplifying expressions with brackets

If we want to subtract 2 from 3 first, and then subtract the result from 10, we write it as $10 - (3 - 2)$, with $3 - 2$ within brackets. This emphasises that the operation within brackets has to be done first.

That is, $10 - (3 - 2) = 10 - 1 = 9$.

Consider the following example.

A practical examination in music is held over six days. Each day, twelve candidates participate in the morning session, while 8 participate in the afternoon session. Let us find the total number of candidates.

Number of candidates in each morning session $= 12$
Number of candidates in each afternoon session $= 8$
Total number of candidates during the six days $= (12 + 8) \times 6$
$= 20 \times 6 = 120$

Observe that the usage of brackets has been necessary in deriving the correct answer.

The convention followed when simplifying expressions involving whole numbers and the operations +, −, ×, ÷, and brackets is as follows.

- First perform any calculations inside brackets.
- Then perform all multiplications and divisions, working from left to right.
- Finally perform all additions and subtractions, working from left to right.

Example 1

Simplify $20 \div (12 - 7)$.

$20 \div (12 - 7) = 20 \div 5 = 4$

Example 2

Simplify $5 \times (10 + 12) \div 11$.

$5 \times (10 + 12) \div 11 = 5 \times 22 \div 11$

$= 110 \div 11 = 10$

Example 3

Simplify $8 + 5 \times (10 + 2) \div 3 - 4$

$8 + 5 \times (10 + 2) \div 3 - 4 = 8 + 5 \times 12 \div 3 - 4$

$= 8 + 60 \div 3 - 4$

$= 8 + 20 - 4$

$= 28 - 4$

$= 24$

Example 4

The pencils in five boxes, each of which contains 12 pencils, are divided equally among 4 students. Write down an expression for the number of pencils a single student receives, and simplify it.

$(12 \times 5) \div 4 = 60 \div 4 = 15$

The number of pencils each child receives is 15.

Example 5

Nimal plucked 47 mangoes from a tree in his garden. He kept 18 in his possession, and sold the rest to his neighbour at Rs. 9 per fruit. Write down an expression for the total amount of money Nimal earned in rupees by selling the mangoes, and simplify it.

$(47 - 18) \times 9 = 29 \times 9 = 261$

This can also be written as $9 \times (47 - 18)$ or as $9\ (47 - 18)$, omitting the multiplication symbol.

The total amount earned by selling the mangoes is 261 rupees.

Example 6

The taxi fare for the first kilometre is Rs. 50. It is Rs. 42 for each kilometre above the first. Write down an expression for the amount paid by a passenger who enjoyed a ride of 12 kilometres. Simplify your expression.

$50 + 42\ (12 - 1) = 50 + 42 \times 11 = 50 + 462 = 512$

The total amount paid is 512 rupees.

Exercise 3.4

(1) Simplify the following expressions.

(i) $(12 + 8) - 15$ (ii) $35 - (14 + 9)$ (iii) $7\ (12 - 7)$

(iv) $108 + 3\ (27 - 13)$ (v) $24 \div (17 - 5)$ (vi) $3\ (5 + 2) \times 8$

(vii) $31 + (16 \div 4)$ (viii) $73 - (8 \times 9)$ (ix) $(19 \times 10) + 38$

(x) $475 - (30 \div 6)$

(2) An international call to a certain country costs Rs. 7 for the first minute, and Rs. 4 per minute thereafter. Write down an expression in rupees for the cost of a 10 minutes long international call. Simplify your expression.

(3) Write down an expression for the number of two-litre bottles that can be filled with a fruit drink made from 8 litres of water and 4 litres of fruit juice.

(4) Simplify the following expressions.

(i) $30 \div 10 \times 5$ (ii) $40 \times 10 \div 5$ (iii) $400 - 20 \times 10$

(iv) $30 \div (10 \times 3)$ (v) $(40 \div 10) \times 8$ (vi) $3 + 7 \times 5$

(vii) $6 \div 2 + 7$ (viii) $(24 \times 3) \div 8$ (ix) $24 \div (3 \times 4)$

(x) $3 + 6 \times (5 + 4) \div 3 - 7$ (xi) $10 + 8\ (11 - 3) \times 4 - 4$

Summary

- The convention followed when simplifying expressions involving whole numbers and the operations +, –, ×, ÷ and brackets is as follows.
- First perform any calculations inside brackets.
- Then perform all multiplications and divisions, working from left to right.
- Finally perform all additions and subtractions, working from left to right.