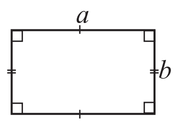
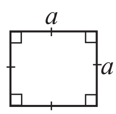
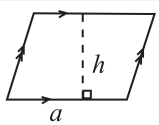
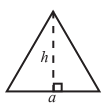
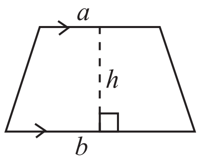
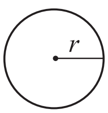
6 Area

By studying this lesson you will be able to

- find the areas of sectors of circles,
- solve problems related to the areas of compound plane figures containing sectors of circles.

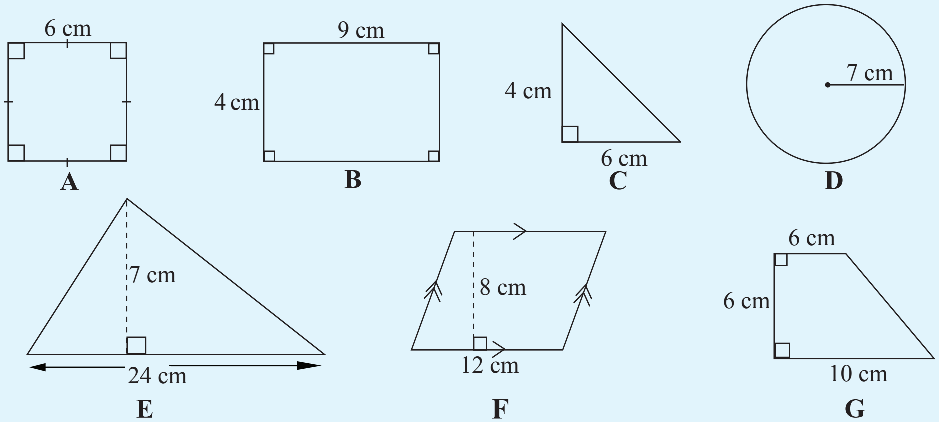
Areas of plane figures

Let us recall some facts you have learnt in previous grades under the topic **Area**.

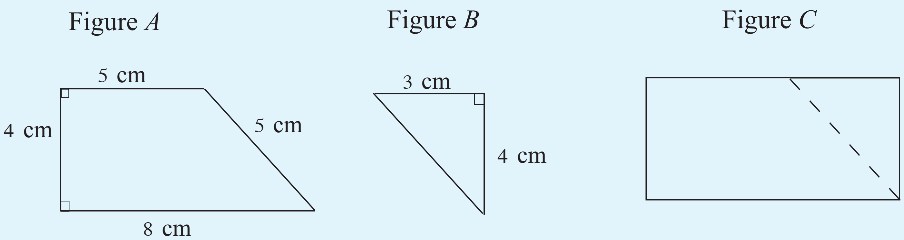
Name	Plane Figure	How the area is calculated	Formula for the area (A)
Rectangle		length × breadth	$A = a \times b$
Square		(length of a side)^2	$A = a^2$
Parallelogram		base × altitude	$A = a \times h$
Triangle		$\frac{1}{2}$ × base × altitude	$A = \frac{1}{2} \times a \times h$
Trapezium		$\frac{1}{2}$ × sum of the lengths of the parallel sides × altitude	$A = \frac{1}{2}(a+b) \times h$
Circle		π × (radius)^2	$A = \pi r^2$

Review Exercise

1. Find the area of each of the following plane figures.



2. The rectangle in Figure *C* has been formed by joining together the trapezium in Figure *A* and the triangle in Figure *B*.

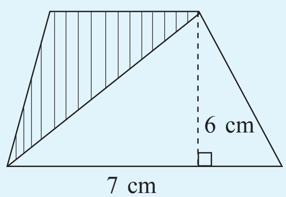


(i) Find the area of the trapezium in Figure *A*.

(ii) Find the area of the triangle in Figure *B*.

(iii) Find the area of the rectangle in Figure *C* in terms of the areas of Figure *A* and Figure *B*.

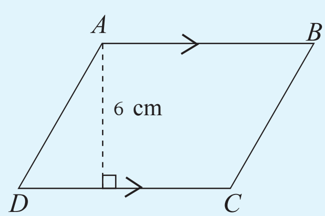
3. The figure denotes a trapezium of area 33 cm^2 that has been formed by joining two triangles together. Find the area of the triangle which is shaded.



4. The figure denotes a parallelogram of area 120 cm^2. Its perimeter is 64 cm. Determine the following based on the information that is given.

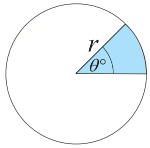
(i) The length of the side *CD*.

(ii) The length of the side *BC*.

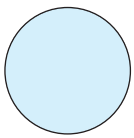
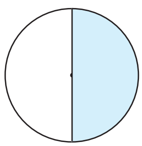
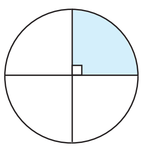
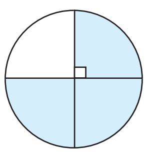
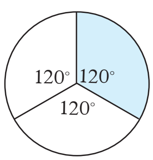
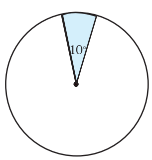
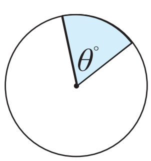


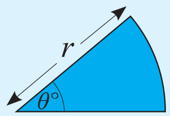
6.1 Area of a sector of a circle

We considered how the perimeter of a sector of a circle is found in the lesson on Perimeters. Now let us consider how the area of a sector of a circle is found.



The following table shows how the area of a sector is found when the angle at the centre of the sector takes certain special values.

Sector	Shaded Sector as a fraction of the circle	Area of the Sector
	1	πr^2
	$\frac{1}{2}$	$\frac{1}{2} \times \pi r^2$
	$\frac{1}{4}$	$\frac{1}{4} \times \pi r^2$
	$\frac{3}{4}$	$\frac{3}{4} \times \pi r^2$
	$\frac{1}{3}$	$\frac{1}{3} \times \pi r^2$
	$\frac{10}{360}$	$\frac{10}{360} \times \pi r^2$
	$\frac{\theta}{360}$	$\frac{\theta}{360} \times \pi r^2$

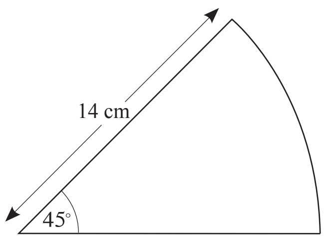


According to the pattern in the table, the area of the sector of radius r and angle at the centre $\theta°$ is $\dfrac{\theta}{360} \times \pi r^2$

Let us consider (through the following examples) how the area of a sector is found using this result. In the examples and exercises of this chapter, the value of π is taken as $\dfrac{22}{7}$.

Example 1

Find the area of the sector in the following figure.



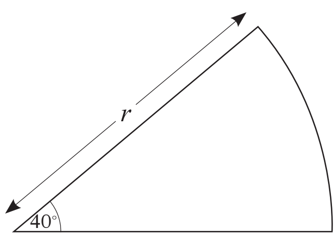
$$\text{Area} = \frac{45}{360} \times \pi r^2$$
$$= \frac{45}{360} \times \frac{22}{7} \times 14 \times 14$$
$$= 77$$

$\therefore$ Area is 77 cm^2.

Example 2

If the area of the sector in the figure is $17\frac{1}{9}\,\text{cm}^2$, find the radius of the corresponding circle.

Let us take the radius as r cm.

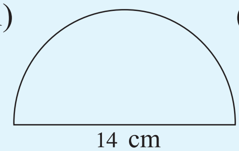


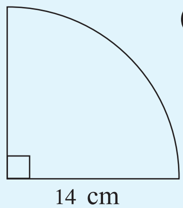
$$\text{Area} = \frac{40}{360} \times \pi r^2$$
$$17\frac{1}{9} = \frac{1}{9} \times \frac{22}{7} \times r^2$$
$$\frac{154}{9} = \frac{1}{9} \times \frac{22}{7} \times r^2$$
$$r^2 = \frac{154 \times 7}{22}$$
$$r = 7$$

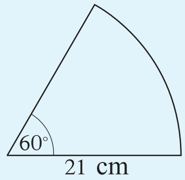
$\therefore$ Radius is 7 cm.

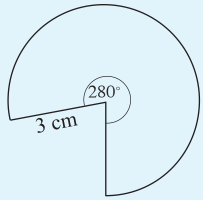
Exercise 6.1

1. Find the area of each sector.

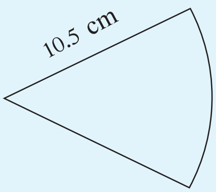
(i)


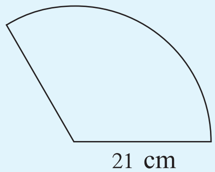
(ii)


(iii)


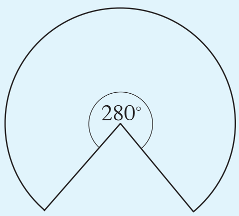
(iv)


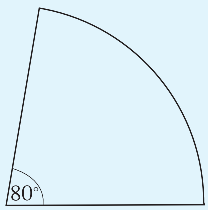
2. The areas of the two sectors of circles given below are 77 cm^2 and 462 cm^2 respectively. Find the angle at the centre of each sector.

(i)


(ii)


3. The areas of the two sectors of circles given below are 792 cm^2 and $6\frac{2}{7}$ cm^2 respectively. For each sector, find the radius of the corresponding circle.

(i)


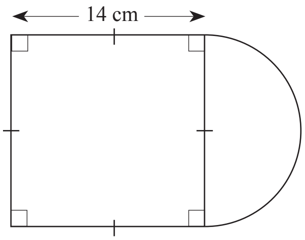
(ii)


6.2 Plane figures containing sectors of circles

Let us consider the areas of plane figures formed by sectors of circles and other simple plane figures such as rectangles and triangles being joined together.

Example 1

The following denotes a plane figure consisting of a square and a semi-circle. Find its area.



Area of the square $= 14 \text{ cm} \times 14 \text{ cm}$

$= 196 \text{ cm}^2$

Since the diameter of the semi-circle is equal to the length of a side of the square,
The radius of the circle is $14 \div 2 = 7$ cm.

Area of the semi-circle $= \frac{1}{2} \times \pi r^2$

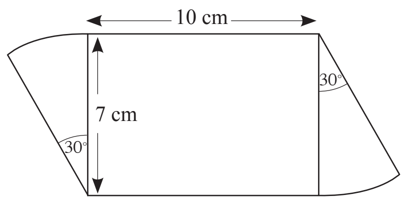
$= \frac{1}{\cancel{2}_1} \times \frac{\cancel{22}^{11}}{\cancel{7}_1} \times \cancel{7}^1 \times 7 = 77 \text{cm}^2$

Area of the compound plane figure $= 196 \text{ cm}^2 + 77 \text{ cm}^2$

$= \underline{\underline{273 \text{ cm}^2}}$

Example 2

The figure denotes a compound plane figure consisting of a rectangle and two sectors of a circle. Find its area.



Area of the rectangle $= 10 \times 7$

$= 70 \text{ cm}^2$

Area of a sector $= \frac{30}{360} \times \pi r^2$

$= \frac{30}{360} \times \frac{22}{7} \times 7 \times 7$

$= \frac{77}{6} \text{cm}^2$

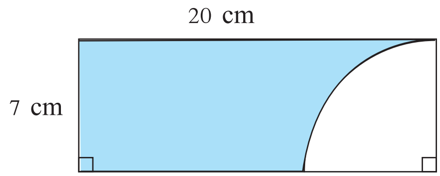
Area of both sectors $= \frac{77}{6} \times 2 = \frac{77}{3} = 25\frac{2}{3} \text{cm}^2$

Area of the compound plane figure $= 70 \text{cm}^2 + 25\frac{2}{3} \text{cm}^2$

$= \underline{\underline{95\frac{2}{3} \text{ cm}^2}}$

Example 3

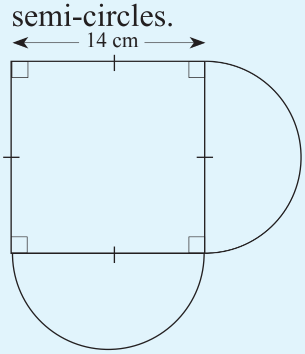
The shaded portion is obtained by cutting $\frac{1}{4}$ of a circle from a rectangular lamina. Find the area of the shaded portion using the given data.



$$\text{Area of the rectangle} = 20 \times 7$$
$$= 140 \text{ cm}^2$$
$$\text{Area of the sector} = \frac{90}{360} \times \pi r^2$$
$$= \frac{90}{360} \times \frac{22}{7} \times 7 \times 7$$
$$= 38.5 \text{ cm}^2$$
$$\therefore \quad \text{Area of the shaded portion} = 140 - 38.5$$
$$= \underline{\underline{101.5 \text{ cm}^2}}$$

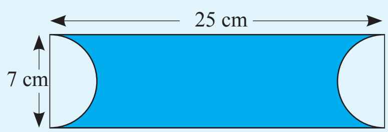
Exercise 6.2

1. The following is a compound plane figure consisting of a square and two semi-circles.



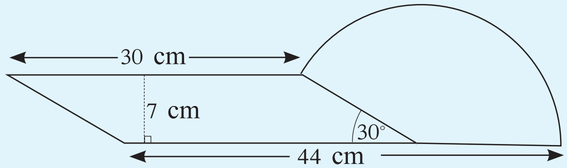
(i) Find the area of the square.
(ii) Find the radius of a semi-circular portion.
(iii) Find the total area of the two semi-circular portions.
(iv) Find the area of the compound plane figure.

2. The shaded portion in the figure was obtained by cutting out two semi-circular parts from a rectangular piece of paper.



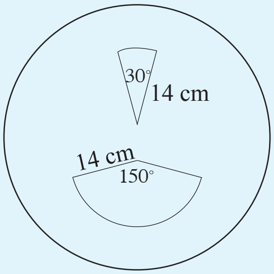
(i) Find the area of the rectangle.
(ii) Find the total area of the two semi-circular parts.
(iii) Find the area of the shaded portion.

3. The following is a compound plane figure consisting of a parallelogram and a sector of a circle.

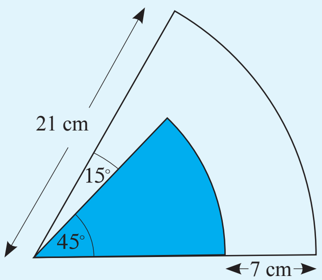


(i) Find the area of the parallelogram.
(ii) Find the area of the sector.
(iii) Find the area of the compound figure.

4. The figure denotes a circular lamina of radius 28 cm. The two sectors in the figure are to be cut out. Find the area of the remaining portion after the two sectors have been cut out.

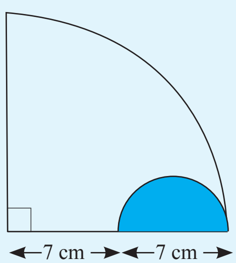


5. The following is a figure consisting of two sectors of circles.



Show that the ratio of the area of the smaller sector to the larger sector is 1 : 3.

6. Based on the measurements given in the figure, show that the area of the unshaded region of the larger sector is seven times the area of the shaded sector.

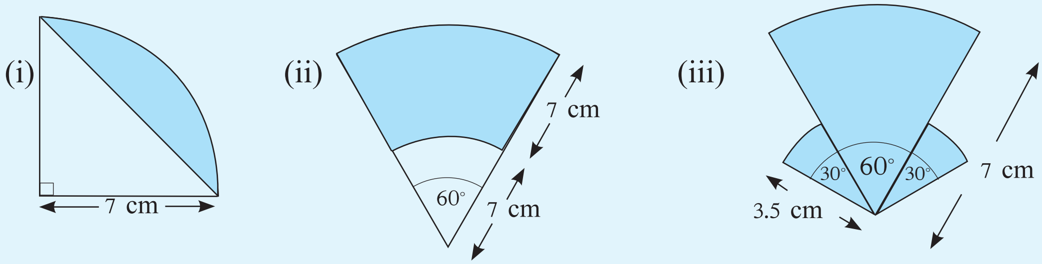


Summary

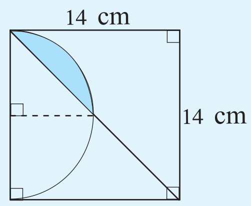
The area of a sector of a circle of radius r with angle at the centre θ° is $\frac{\theta}{360} \times \pi r^2$.

Miscellaneous Exercise

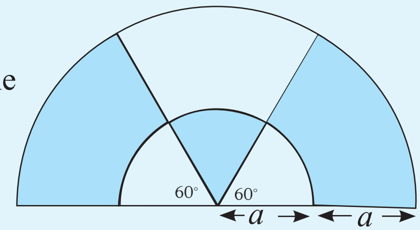
1. Find the area of the shaded portion in each of the figures given below which are formed by sectors of circles.



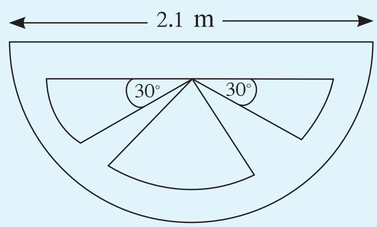
2. Find the area of the shaded region.
The curved line in the figure is the arc of a semi-circle.



3. The figure denotes two semi-circles.
Show that the ratio of the un-shaded region to the shaded region in the figure is 5 : 7.



5. A sketch of the area in front of a commemorative plaque is given in the figure. Grass has been grown in the three portions within the semi-circle which are in the shape of sectors of circles, while white sand has been spread in the other regions. The radius of each sector within the semi-circle is 84 cm.



(i) What is the radius of the semi-circle in centimetres?
(ii) Find the area of the semi-circular part in cm^2.
(iii) Find the area of one of the sectors with angle at the centre equal to 30^o.
(iv) Find the angle at the centre of the large sector if its area is 1848 square centimetres more than the sum of the areas of the other two sectors.