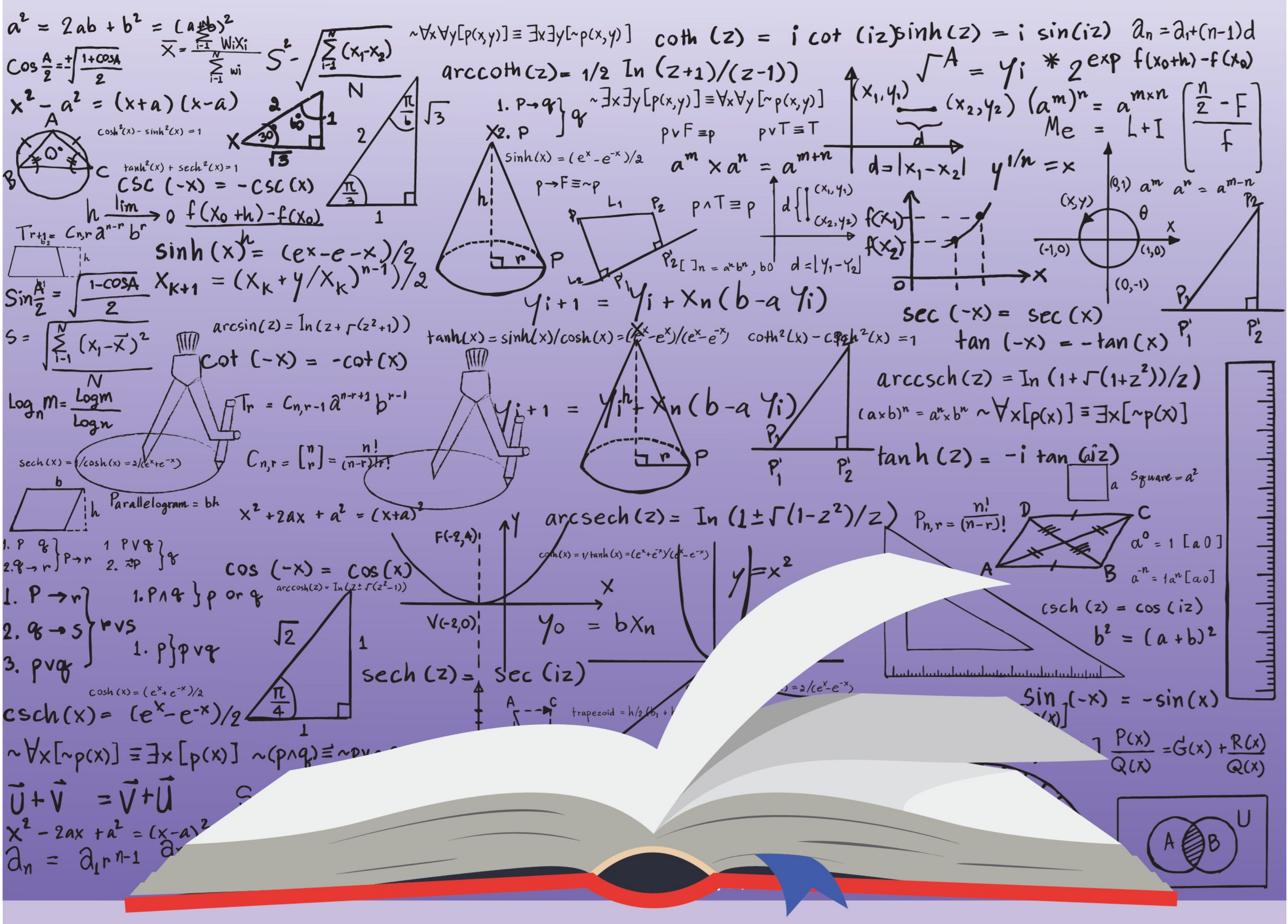
தொகையீடு



தேர்ச்சி: 16.7

பகுதிகளாகத் தொகையிடல்

$$\int uv\,dx = u\left(\int v\,dx\right) - \int\left(\int v\,dx\right)\frac{du}{dx}.dx$$

$$\frac{d}{dx}(uw) = u\frac{dw}{dx} + w\frac{du}{dx}$$

$$u\frac{dw}{dx} = \frac{d}{dx}(uw) - w\frac{du}{dx}$$

$$\int u\frac{dw}{dx}dx = \int\frac{d}{dx}(uw)dx - \int w\frac{du}{dx}.dx$$

$$\int u\frac{dw}{dx}.dx = uw - \int w\frac{du}{dx}.dx \longrightarrow (1)$$

$$v = \frac{dw}{dx}$$

$$dw = v\,dx$$

$$\int dw = \int v\,dx$$

$$w = \int v\,dx$$

$$(1) \Rightarrow \int uv\,dx = u\left(\int v\,dx\right) - \int\left(\int v\,dx\right)\frac{du}{dx}.dx$$

உதாரணம்

01

$$\int xe^{x}dx$$

$$= xe^{x} - \int e^{x}.1.dx$$

$$= xe^{x} - e^{x} + c$$

02

$$\int xe^{2x}dx$$

$$= x\frac{e^{2x}}{2} - \int\frac{e^{2x}}{2}.1.dx$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{2}\frac{e^{2x}}{2} + c$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c$$

$$= \frac{1}{4}e^{2x}(2x-1) + c$$

03

$$\int_0^1 x^2 e^x dx$$

$$= \{x^2 e^x\}_0^1 - \int_0^1 e^x .2x.dx$$

$$= e - 2\int_0^1 xe^x d^x$$

$$= e - 2\left\{[xe^2]_0^1 - \int_0^1 e^x dx\right\}$$

$$= e - 2(e-0) + 2\{e^x\}_0^1$$

$$= -e + 2\{e-1\}$$

$$= e - 2$$

04

$$\int xSinx.dx$$

$$= x(-Cosx) - \int (-Cosx).1dx$$

$$= -xCosx + \int Cosxdx$$

$$= -xCosx + Sinx + c$$

05

$$\int x^2 Sinx.dx$$

$$= x^2(-Cosx) - \int (-Cosx).2x.dx$$

$$= -x^2 Cosx + 2\int Cosx.dx$$

$$= -x^2 Cosx + 2\left\{x.Sinx - \int Sin.x.1.dx\right\}$$

$$= -x^2 \quad Cosx + 2x \quad Sinx + 2Cosx + c$$

06

$$\int xSec^2 x.dx$$
$$= x\tan x - \int \tan x.1.dx$$
$$= x\tan x + \int \frac{-Sin}{Cosx}.dx$$
$$= x\tan x + \ell n|Cosx| + c$$

07

$$\int x^n(\ell nx)dx$$
$$= (\ell nx).\frac{x^{n+1}}{n+1} - \int \frac{x^{n+1}}{n+1}.\frac{1}{x}dx$$
$$= \frac{x^{n+1}}{n+1}\ell nx - \frac{1}{n+1}\int x^n dx$$
$$= \frac{x^{n+1}}{n+1}\ell nx - \frac{1}{(n+1)}.\frac{x^{n+1}}{(n+1)} + c$$
$$= \frac{x^{n+1}}{n+1}\ell nx - \frac{x^{n+1}}{(n+1)^2} + c$$

08

$$\int x(\ell nx)^2.dx$$
$$= (\ell nx)^2.\frac{x^2}{2} - \int \frac{x^2}{2}.2(\ell nx)^1.\frac{1}{x}.dx$$
$$= \frac{x^2}{2}(\ell nx)^2 - \int x\ell nx.dx$$
$$= \frac{x^2}{2}(\ell nx)^2 - \left\{(\ell nx)\frac{x^2}{2} - \int \frac{x^2}{2}.\frac{1}{x}.dx\right\}$$
$$= \frac{x^2}{2}(\ell nx)^2 - \frac{x^2}{2}(\ell nx) + \frac{1}{2}\int xdx$$
$$= \frac{x^2}{2}(\ell nx)^2 - \frac{x^2}{2}(\ell nx) + \frac{1}{4}x^2 + c$$

09

$$\int 1.\tan^{-1} x.dx$$
$$= \left(\tan^{-1} x\right)x - \int x.\frac{1}{1+x^2}dx$$
$$= x\left(\tan^{-1} x\right) - \frac{1}{2}\int \frac{2x}{1+x^2}dx$$
$$= x\left(\tan^{-1} x\right) - \frac{1}{2}\ell n\left(1+x^2\right) + c$$

10

$$\int x^2 \tan^{-1} x.dx$$
$$= \tan^{-1} x.\frac{x^3}{3} - \int \frac{x^3}{3}.\frac{1}{\left[1+x^2\right]}.dx$$
$$= \frac{x^3}{3}\tan^{-1} x - \frac{1}{3}\int \frac{x\left(x^2+1\right)-x}{x^2+1}.dx$$
$$= \frac{x^3}{3}\tan^{-1} x - \frac{1}{3}.\frac{x^2}{2} + \frac{1}{6}\log\left(x^2+1\right) + c$$

11

$$\int_1^{e^\pi} Sin(\ell nx)dx$$
$$I = \int_1^{e^\pi} 1.Sin(\ell nx).dx$$
$$= \{Sin(\ell nx).x\}_1^{e^\pi} - \int_1^{e^\pi} xCos(\ell nx).\frac{1}{x}.dx$$
$$= -\int_1^{e^\pi} Cos(\ell nx).dx$$
$$= -\left\{(Cos(\ell nx).x)_1^{e^\pi} - \int_1^{e^\pi} x(-)Sin(\ell nx).\frac{1}{x}.dx\right\}$$
$$= -\{-e^\pi - 1\} - I$$
$$I = \frac{1}{2}\left(e^\pi + 1\right)$$

12

$$\int e^{\pi} Sinx.dx$$

$$I = \int e^{\pi} Sinx.dx$$

$$= e^{x}(-Cosx) - \int (-Cosx)e^{x}.dx$$

$$= -e^{x}Cosxt\int e^{x}Cosx.dx$$

$$= -e^{x}Cosxt\left\{e^{x}.Sinx - \int Sinx.e^{x}.dx\right\}$$

$$= e^{x}(Sinx - Cosx) - I + c$$

$$2I = e^{x}(Sinx - Cosx) + c$$

$$I = \frac{e^{x}}{2}(Sinx - Cosx) + c^{l}$$ இங்கு $c^{l} = \frac{c}{2}$

முறை II (Method II)

12

$$I = \int e^{x} Sinx.dx$$

$$= Sinx.e^{x} - \int e^{x}Cosx.dx$$

$$= e^{x}Sinx - \left\{Cos.e^{x} - \int e^{x}(-Sinx)dx\right\}$$

$$= e^{x}(Sinx - Cosx) - I + c$$

$$2I = e^{x}(Sinx - Cosx) + c$$

$$I = \frac{e^{x}}{2}(Sinx - Cosx) + c^{l}$$ இங்கு $c^{l} = \frac{c}{2}$

13

$$\int x(\tan x)^{1} Sec^{2}x.dx$$

$$= x\frac{(\tan x)^{2}}{2} - \int \frac{(\tan x)^{2}}{2}.1.dx$$

$$= \frac{x}{2}\tan^{2}x - \frac{1}{2}\int (Sec^{2}x - 1)dx$$

$$= \frac{x}{2}\tan^{2}x - \frac{1}{2}\{\tan x - x\} + c$$

14

$$\int xSin^2x.Cosx.dx$$
$$= \int x(Sinx)^2 Cosx.dx$$
$$= x\frac{(Sinx)^3}{3} - \int \frac{(Sinx)^3}{3}.1.dx$$
$$= x\frac{Sin^3x}{3} + \frac{1}{3}\int Sin^2x.d(Cosx)$$
$$= x\frac{Sin^3x}{3} + \frac{1}{3}\int (1 - Cos^2x)d(Cosx)$$
$$= x\frac{Sin^3x}{3} + \frac{1}{3}\left(Cosx - \frac{Cos^3x}{3}\right) + c$$

15

$$\int xSinx\ Cosx.dx$$
$$= \frac{1}{2}\int xSin2x.dx$$
$$= \frac{1}{2}\left\{x\left(\frac{-Cos2x}{2}\right) - \int \frac{-Cos2x}{2}.1.dx\right\}$$
$$= \frac{-x}{4}Cos2x + \frac{1}{4}\int Cos2x.dx$$
$$= -\frac{x}{4}Cos2x + \frac{1}{4}.\frac{Sin2x}{2} + c$$
$$= -\frac{x}{4}Cos2x + \frac{1}{8}Sin2x + c$$

பிரதியிடலுடன் கூடிய பகுதிகளாகத் தொகையிடல்

01

$$I = \int Cos\sqrt{x}.dx$$

$t = \sqrt{x}$ என்க.

$$\frac{dt}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{2t}$$

$$I = \int Cost.2t.dt$$

$$= 2\left\{tSin.t - \int Sin.t.1.dt\right\}$$

$$= 2t.Sin.t + 2Cos.t + c$$

$$= 2\sqrt{x}Sin.\sqrt{x} + 2Cos\sqrt{x} + c$$

02

$$I = \int x^2 Sin(x^2)dx$$

$$t = x^2 \Rightarrow \frac{dt}{dx} = 2x$$

$$I = \int x^2 Sin(x^2).xdx$$

$$= \int t.Sin.t.\frac{dt}{2}$$

$$= \frac{1}{2}\left\{t(-Cost) - \rightarrow \int (-Cost).1.dt\right\}$$

$$= -\frac{1}{2}tCost + \frac{1}{2}Sin.t + c$$

$$= -\frac{1}{2}x^2Cosx^2 + \frac{1}{2}Sinx^2 + c$$

03

$$I=\int_0^1 \left(Sin^{-1}x\right)^2.dx$$

$$\theta=Sin^{-1}x \Rightarrow Sin\theta=x \Rightarrow \frac{dx}{d\theta}=Cos\theta$$

$$x=0 \Rightarrow \theta=0$$

$$x=1 \Rightarrow \theta=\frac{\pi}{2}$$

$$I=\int_0^{\frac{\pi}{2}} \theta^2 Cos\theta.d\theta$$

$$=\left\{\theta^2 Sin\theta\right\}_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}} Sin\theta.2\theta.d\theta$$

$$=\left(\frac{\pi}{2}\right)^2-2\left\{\left(\theta\left(-Cos\theta\right)\right)_0^{\pi/2}-\int_0^{\pi/2}-Cos\theta.1.d\theta\right\}$$

$$=\frac{\pi^2}{4}-2\left\{0+\int_0^{\pi/2}Cos\theta.d\theta\right\}$$

$$=\frac{\pi^2}{4}-2\left\{Sin\theta\right\}_0^{\pi/2}$$

$$=\frac{\pi^2}{4}-2\{1-0\}$$

$$=\frac{\pi^2}{4}-2$$

04

$$I=\int x^2 Sin^{-1}x.dx$$

$$\theta=Sin^{-1}x \Rightarrow Sin\theta=x \Rightarrow \frac{dx}{d\theta}=Cos\theta$$

$$I=\int Sin^2\theta.\theta.Cos\theta.d\theta$$

$$=\int \theta(Sin\theta)^2.Cos\theta.d\theta$$

$$=\theta.\frac{(Sin\theta)^3}{3}-\int \frac{(Sin\theta)^3}{3}.1.d\theta$$

$$=\theta\frac{Sin^3\theta}{3}+1+\frac{1}{3}\int Sin^2\theta.d.(Cos\theta)$$

$$=\frac{\theta}{3}.\frac{Sin^3.\theta}{3}+\frac{1}{3}\int Sin^2\theta.d.(Cos\theta)$$

$$=\frac{\theta}{3}Sin^3\theta+\frac{1}{3}\int(1-Cos^2\theta)d(Cos\theta)$$

$$=\frac{\theta}{3}Sin^3\theta+\frac{1}{3}\left\{Cos\theta-\frac{Cos^3\theta}{3}\right\}+c$$

$$=(Sin^{-1}x)\frac{x^3}{3}+\frac{1}{3}\left\{\sqrt{1-x^2}-\frac{(1-x^2)^{3/2}}{3}\right\}+c$$

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தொகுப்பு :-திரு மு.ராகுலன், ஆசிரியர் இணைந்த கணிதம் (யா/மகாஜனா கல்லூரி)
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